IMPROVING THE OUTLIER RESISTANCE AND ACCURACY OF PRISM: DESCRIPTION AND SAMPLE ANALYSIS

Clifford M. Hurvich* and Margaret F. Fels

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Center for Energy and Environmental Studies
The Engineering Quadrangle
Princeton University
Princeton, NJ 08544

*Current address: Department of Statistics and Operations Research, New York University, 40th W. 4th Street, New York, NY, 10003.

Abstract

It is well known that outliers (unusual or stray values) can and often do occur in actual consumption data. PRISM (PRInceton Scorekeeping Method), a statistical procedure which estimates weather-normalized energy consumption from actual consumption data, often has only twelve or fewer (monthly) data points and thus is susceptible to the influence of outliers. In this paper, enhancements of the PRISM model are developed to reduce the impact of outliers on the PRISM estimates.

The result is a robust version of PRISM, or Robust PRISM, which identifies and downweights outliers in the consumption data, and which gives results that are identical to those obtained from "ordinary" PRISM in the absence of outliers. A related feature of the method is the ability to adjust for unequal period lengths; the result is Weighted PRISM. In keeping with the objective that Robust PRISM be a direct extension of PRISM, a robust version of the R^2 statistic is developed. The formulae for Robust PRISM, whose theoretical underpinnings are developed and tested for this study, are natural extensions of the original formulae used in PRISM.

The performance of Robust PRISM is compared with PRISM by application to consumption data for several sets of houses. Electricity consumption data are used to test the method's resistance to outliers, and oil consumption data are used to examine the method's ability to adjust for unequal period lengths. Although the methodology is in a research stage of development, the results reported here illustrate the value of robustness in the PRISM model and the improvements that can ensue.

Note: To facilitate continued review of this methodology development, extremely detailed results are included in some of the tables and figures. Those tables and figures containing concise summaries of these results are indicated by asterisks in the text. For a cursory reading of the text, the reader may wish to emphasize the (asterisked) summaries.

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TABLE OF CONTENTS

List of Tables

* Summary table (see note with abstract).

List of Figures

I. INTRODUCTION

Increasing attention is being paid to the evaluation of energy savings based on <u>actual</u> energy consumption measurements. Actual consumption data, from gas or electricity meter readings or from fuel oil deliveries, tell the truth about how much energy has been saved, in a way that models based on simulated data cannot. While the structural component of a building's energy consumption may be accurately modeled, the effect of the occupants of the building on the energy consumption is far less quantifiable. Thus real-world data are essential for understanding how much energy has actually been saved.

PRISM, the PRInceton Scorekeeping Method, is a statistical procedure which uses actual consumption data to estimate weather-normalized energy consumption and savings. Generally applied to one year of approximately monthly data from before and after the conservation action of interest, PRISM has been found to be a productive tool for monitoring savings in all major fuel types (Fels, ed., 1986). Its main problem is its sensitivity to outliers (unusual or stray values) that often occur in actual consumption data; when there are few data points (12 or less, as is often the case in PRISM), and when the statistical procedure is based on least-squares regression (as it is in PRISM), a single outlier can have an undesirably strong influence on the results. A second problem is that of unevenly spaced meter readings which occur frequently with oil deliveries, for example. Both of these problems are inevitable artifacts of real-world data. The first problem especially will be encountered as long as people continue to live or work in the buildings being monitored.

In this paper, enhancements of PRISM are developed to reduce the impact of these problems on the PRISM estimates. Most of the work is devoted to development of a robust version of PRISM, to reduce the effect of outliers. From that, a version of PRISM to compensate for unequal period lengths naturally follows. Before describing the new work, a brief synopsis of the original PRISM method is in order. For more detail, the reader is referred to an introductory paper on PRISM (Fels, 1986).

I.A. Synopsis of Ordinary PRISM

Three physical parameters result from PRISM applied to whole-house billing data for the heating fuel of an individual house (or building): base-level consumption α, providing a measure of the appliance (temperature-independent) usage in the house; reference temperature τ, corresponding to the average outdoor temperature above which no fuel is required for heating; and the heating slope β, giving the amount of fuel required for each incremental drop in outdoor temperature below τ. The house's index of consumption, NAC, or Normalized Annual Consumption, is obtained from these three parameters applied to a long-term annual (i.e., "typical year") average of heating degree-days.

The two data requirements for each period i included in the analysis are average daily consumption, F_i, and heating degree-days per day, $H_i(\tau)$, computed to reference temperature τ. F_i is generally computed from meter readings, and $H_i(\tau)$ from average daily temperatures from a nearby weather station (see Fels, 1986).

In ordinary PRISM (the version of PRISM currently being used by over 150 groups in the U.S. and other countries), the three parameters are found by a least-squares fit of the set of data points $\{F_i\}$ and $\{H_i\}$ to a linear model:

$$F_i = \alpha + \beta H_i(\tau) + \epsilon_i \qquad (1)$$

where ϵ_i is the random error term. "Best τ" is found as the value of τ for which a plot of F_i vs. $H_i(\tau)$ is most nearly a straight line, or, formally in ordinary least-squares linear regression, for which the R^2 statistic is highest. The corresponding values of α and β are the best estimates of base level and heating slope. NAC is then determined from

$$\text{NAC} = 365\alpha + \beta H_0(\tau) \quad (2)$$

where $H_0(\tau)$ is the heating degree-days (base τ) for the typical year.

The NAC estimate provides a reliable consumption index from which energy savings may be accurately estimated. As shown in Table 1, its standard error is typically only 3-4% of the estimate. On the other hand, the individual parameters, β and τ, and also $\beta H_0(\tau)$, the heating part of NAC, are considerably less well determined, as the larger standard errors in Table 1* show. They are much more sensitive than NAC is to outliers or to the choice of which months were included in the estimation. It is the resulting instability of the individual PRISM parameters that motivates this work.

I.B. <u>The Need for a Robust Version of PRISM</u>

Outliers in energy consumption data arise either from corrupted data (e.g., incorrect meter readings, typing/transcription errors, etc.) or from accurate but atypical data (e.g., brief vacations, sporadic cooling, other temporary changes). Regardless of the cause, such outliers tend to have an adverse effect on the quality of the resulting PRISM estimates. Although the estimates of NAC seem to be relatively insensitive to outliers, the estimates of the individual parameters, α, β, and τ, can be strongly influenced by even a single bad point. Such an extreme sensitivity to outliers is undesirable: we would like our estimates to reflect the behavior of the bulk of the data, instead of being pulled away to accommodate a few unusual values.

One reasonable strategy for alleviating the problem is to identify the outliers (by inspecting plots of residuals, for example), and then to re-run the PRISM program with the outlier removed. This approach, however, has at least two substantial drawbacks. First, outlier identification is not as simple as one might think, particularly since PRISM tends to adjust the

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An asterisk indicates a Table or Figure with summary results (e.g., Table 6, which summarizes the SAS Univariate output in Figures 2-5).

reference temperature to bring outliers as much in line as possible with the other points in the consumption vs. heating degree-day plot. Second, even if we could devise a method of deciding which points are outliers, it is not clear that we should simply throw away all of these points. It seems wasteful, for example, to discard points completely which are only slightly beyond some arbitrary cutoff value that defines them as outliers, especially since we often start with only 10 or 12 data points.

In our opinion, the problems caused by outliers, as well as the drawbacks of the expedient solution described above, warrant the development of a modified version of PRISM, which we call Robust PRISM, or RPRISM. It is specifically designed with three objectives: 1) to be robust (i.e., insensitive to outliers), without completely discarding any of the available information; 2) to give results which agree with those of PRISM itself when the data do not contain outliers; and 3) to take advantage of the years of development and application of the PRISM algorithm.

In Section II, we describe the RPRISM algorithm in fairly general terms, deferring the details to Appendix A, and we explain how RPRISM yields the useful by-product of automatic outlier identification. As we will show, RPRISM can be interpreted as automatically deciding how much weight to assign to each point, with the lowest weights going to the outliers. (Currently, PRISM computes ordinary unweighted least squares estimates.) Next, in Sections IIIA and IIIB we compare the performance of PRISM and RPRISM for two data sets. Each set consists of 50 electrically heated houses in New Jersey. The houses in the first set have no air conditioning, and the second have central air conditioning. The sporadic cooling found in many houses in heating-dominated climates has long been recognized as a nuisance to the heating-only PRISM method. The cooling points often appear as outliers in the consumption vs. heating degree-day plots, and tend to inflate the base-level estimate α. Therefore, the second data base offers RPRISM the opportunity to remove some of the problems previously encountered with regular least-squares PRISM.

I.C. <u>The Need for a Weighted Version of PRISM</u>

Although not directly related to robustness, another problem with the PRISM approach is that PRISM gives equal weight (i.e., attaches equal importance) to all consumption values, regardless of the relative lengths of the corresponding periods. In the case of oil-heated homes, for which the periods between deliveries in the summer are typically much longer than those in the winter, this can lead to a lack of summer points and a correspondingly poor determination of the base level α. As we will describe in Section IV, statistical considerations suggest that it is sensible to take explicit account of the differing period lengths by weighting each point in proportion to the corresponding period length. This can be accomplished by extending PRISM to compute weighted least-squares estimates. Fortunately, the weighted least-squares (WPRISM) algorithm is easily created by making fairly minor alterations in the basic PRISM algorithm, and WPRISM is in fact one of the key elements in the fully robust version, RPRISM. Thus, the ability to handle unequal period lengths (or, in general, unequal error variances) comes as an additional benefit from the development of RPRISM.

In Section IV.A, we analyze a set of 69 oil-heated houses to examine the effectiveness of the correction for differing period lengths suggested above. Complete specifications of the new algorithms as well as pertinent mathematical derivations are given in Appendix A. These algorithms have been incorporated in a version of PRISM used for research at Princeton. The resulting software is described in Appendix B. In Appendix C, we discuss an additional possible application of the weighted PRISM (WPRISM) algorithm.

II. AN OVERVIEW OF THE RPRISM ALGORITHM

The key to robustifying PRISM lies in adapting the basic method to allow for weighted least squares, in which the estimates of α, β, and τ are chosen to minimize the sum of weighted squared residuals.

$$\sum_{i=1}^{n} w_i \, (Y_i - [\alpha + \beta X_i(\tau)])^2 \tag{3}$$

where $w_1, \ldots, w_n$ is a sequence of fixed nonnegative weights. The weighted least-squares algorithm is called weighted PRISM, or WPRISM. (In Appendix AIV, we describe the mathematical details of computing these WPRISM estimates.) Setting all weights to 1.0 and obtaining estimates to minimize Eq.(3) gives the basic PRISM method. Alternatively, if we give certain outlying points small weights while leaving the remaining weights at 1.0, then the WPRISM estimates will be less influenced by the outliers, and hence more robust, than the ordinary least-squares PRISM estimates. Further, if we allow the size of each weight to be determined by the severity of the outlier, we can progressively decrease the influence of a given outlier according to its severity. Thus, no point need be discarded (given zero weight), but progressively less attention will be paid to increasingly anomalous values.

The fully robust algorithm, RPRISM, proceeds automatically and iteratively, using WPRISM at each stage. A schematic is given in Figure 1.* The first step is to obtain the ordinary least squares (i.e., PRISM) estimates, $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$, as well as the residuals

$$r_i = y_i - [\hat{\alpha} + \hat{\beta}\, x_i\, (\hat{\tau})] \quad \text{for} \quad i = 1, \ldots, n. \tag{4}$$

Next, a set of weights $\{w_i\}$ is determined by the residuals $\{r_i\}$, through a simple formula. (See Appendix AII, step 4 for details.) The formula provides an automatic and objective way of downweighting the effect of outliers. These weights are then used to obtain new provisional estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ by WPRISM. The RPRISM method proceeds iteratively, obtaining new provisional estimates at each stage by WPRISM, and then using the residuals from this fit to compute a new set of weights for use in the next stage. The algorithm converges after a few iterations (i.e., the provisional estimates at a given stage will be almost identical to the ones obtained at the previous stage), yielding the final robust estimates for α, β, and τ.

The final set of weights can be used for outlier detection: the outliers are the points with the smallest weights. Thus, RPRISM provides us not only with robust parameter estimates, but with an automatic outlier identification method as well.

If the final weights are all equal to 1.0, then the PRISM and RPRISM estimates will be identical. This situation, however, is relatively rare. More commonly, most of the weights are 1.0, and the remaining weights are reasonably large (e.g., ≥ 0.6). The resulting RPRISM estimates in this case typically differ only slightly from the PRISM estimates. Another possibility is that one or more of the weights is quite small (e.g., ≤ 0.4), indicating the presence of severe outliers. In this situation, the PRISM and RPRISM estimates can be quite different. More important, as will be shown, the RPRISM estimates in such cases often exhibit much more reasonable behavior than the corresponding PRISM estimates, since the latter are not resistant to the effects of outliers.

To preserve continuity with ordinary PRISM, we have developed a robust version of the R^2 statistic for use with RPRISM. In ordinary PRISM, $R^2(\tau)$ measures the percentage of data variability which can be explained by the fitted model at a given value of τ, and $R^2(\tau)$ is maximized by $\tau = \hat{\tau}$, the least-squares estimate. In RPRISM, for a given iteration of weighted least squares, the new statistic $R^2_W(\tau)$ is maximized by $\tau = \hat{\tau}_W$, the (provisional) weighted least-squares estimate. When used with the final set of weights, R^2_w gives a robust measure of the percentage of explained variability. If the weights are all equal to 1.0, then R^2_w and R^2 will be identical, but if there are one or more strong outliers, R^2_w can be considerably larger than R^2.

Approximate standard errors for the RPRISM estimates can be easily calculated. (See Appendix AIII for details.) The formulae used for the RPRISM standard errors can be justified from a theoretical point of view, and in addition they can be seen to be natural extensions of the formulae used in PRISM. Once again, when the final weights are all 1.0, the PRISM and RPRISM standard errors will be identical. Furthermore, where there are no severe outliers, the standard errors are reasonably similar. Finally, when there are one or more severe outliers, the RPRISM standard errors tend to be much smaller than those given by PRISM, indicating that in this case the RPRISM parameter estimates are much more reliable than the corresponding PRISM estimates.

III. SAMPLE ANALYSES OF ROBUST VS. ORDINARY PRISM

In the next two sections, we will compare the performance of PRISM and RPRISM for two sets of 50 electrically heated houses in New Jersey, one set reporting no air conditioning (Elec-HO), and the other reporting central air conditioning (Elec-AC). These sets of houses were studied previously in an adaptation of PRISM to houses with cooling (Stram and Fels, 1986). The consumption data for each of these houses consist of 12 monthly electricity bills from December 1978 to December 1979. The temperature data are from the Newark weather station (NOAA, 1970-81), and the normalized heating degree-days are based on the standard 12-year normalization period (1970-81) used in other PRISM studies done by Princeton researchers.

III.A. <u>A Data Base of 50 Electrically Heated Homes without Air Conditioning</u>

III.A.1 Comparison of PRISM and RPRISM

Several tables summarize the comparison of RPRISM with PRISM estimates for the Elec-HO sample. Tables 2 and 3 give the house-by-house results for PRISM and RPRISM, respectively. Table 4* gives the mean and median values of α, β, NAC, the heating part of NAC (i.e., $\beta H_0(\tau)$), R^2, and the coefficients of variation for the parameters. Table 5 gives the percent differences for each estimate for all houses, defined as

% Diff = 100(RPRISM Estimate - PRISM Estimate) / PRISM Estimate.

Figures 2-5 give SAS Univariate summaries of the corresponding distributions. The mean, median, quartiles and extremes of the percent differences are given in Table 6.* The quartiles are plotted in Figure 6.*

The percent differences in any given parameter are typically quite small. For example, half of all the percent differences in α were between -0.3% and +0.8%. On the other hand, some notable exceptions are provided by houses J19, J23, J43, J45, and J47, which have differences in α of 171, -15, -12, -10, and 7%, respectively. Later, we will examine these houses in

* See footnote on page 3.

detail, and we will see that the discrepancies between PRISM and RPRISM can be attributed to outliers. It is significant that, for individual houses, the percent differences in NAC tend to be less than those in the other parameters. This is particularly true in houses for which one or more of the percent differences is large. This observation lends support to the finding that the PRISM NAC estimates are reasonably robust, while the PRISM estimates of the remaining parameters are definitely not robust.

In addition to analyzing differences between the PRISM and RPRISM estimates, we should also examine the differences between the standard errors provided by the two methods. A useful quantity for this purpose is the ratio of the PRISM and RPRISM coefficients of variation, which we call CVR, where CVR = CV(PRISM estimate)/CV(RPRISM estimate). (The coefficient of variation, CV, of a parameter is defined as the ratio of the standard error of the parameter, to the parameter value itself.) If CVR is greater than 1.0 for a parameter for a given house, then we can conclude that the RPRISM estimate was better determined (i.e., more precise) than the corresponding PRISM estimate.

Table 7 gives the by-house CVRs for α, β, NAC, and the heating part of NAC. Figures 7-10 give summaries of the distributions of the CVRs. Table 8* gives the mean, median, quartiles and extremes of the CVRs for these same estimates. The corresponding quartiles are plotted in Figure 11.* Table 7 shows that, for a given house, the CVRs are fairly uniform across parameters. Comparison of Tables 7 and 5 shows that large CVRs often (but not always) correspond to large percent differences in the parameter estimates. The stemleaf diagrams of Figures 7-10 are all similar, since the rows of Table 7 are relatively constant. It therefore suffices to focus on NAC, as shown in Figure 9. The median CVR of this distribution is 1.11, while the mean CVR (1.31) is substantially larger. The discrepancy between the mean and median is due to the skewness of the distribution: many ratios are reasonably far above 1.0, but no ratios are far below one. The interpretation is that the RPRISM NAC estimates are in many cases much better determined than the PRISM NAC estimates, and that the reverse situation has not occurred for any of the cases studied. The same phenomenon holds for the other parameters.

Next, we compare the PRISM and RPRISM results for the houses which show the largest difference between the two methods. These houses are J19, J43, J45 and J47.

House J19 provides an example of the dramatic improvement robust PRISM can offer. The results are summarized in Table 9. The PRISM estimates are rather ill-behaved: the estimate for τ is high, at 82^{o}F, and the standard errors for α, τ, and the heating part of NAC are quite large. The RPRISM estimate for τ is down to 75.4^{o}F, still a bit high, but with a standard error reduced from 13 to 2^{o}F. The CVRs are relatively high (24.7, 2.6, 3.1, and 7.6 for α, β, NAC, and the heating part of NAC, respectively), indicating that the RPRISM estimates are much better determined than the corresponding PRISM estimates.

A look at outlying consumption data in the consumption and residuals plots for this house is instructive. Figures 12 and 13 give the consumption vs. heating degree-days (HDD) and residuals vs. HDD plots for PRISM, while Figures 14 and 15 give the analogous plots for RPRISM. Examination of the RPRISM plots shows that points H,E,C, and L are substantial outliers. The primary cause of the "high-TAU" difficulty in PRISM for this house is that the consumption for period H (Jul 20-Aug 19, 1979) is quite low -- lower than any other consumption point. The basic PRISM method adjusts τ to make the consumption vs. HDD plot as straight as possible. Indeed, in Figure 12 from PRISM, with $\tau = 82^{o}$ F, point H is somewhat more in line with the other points than it is in Figure 14 from RPRISM, with $\tau = 75^{o}$F. In addition, since point

H (as the lowest consumption point) is from the hottest summer month, PRISM finds a very high τ so that H lies to the left of the other summer points. Thus, the fact that the PRISM parameters are estimated by least squares forces the intercept (α) of the best line to be unreasonably low, in order to accommodate this single point. In RPRISM, by contrast, point H is essentially ignored (Final Weight = 0.15), and the resulting τ and α are more reasonable. Note that RPRISM also pays little attention to two other outliers, points E and L (Final Weights = 0.13 and 0.14, respectively).

Comparison of the residual plots (Figures 13, 15) shows that RPRISM has succeeded in decreasing the scatter of the bulk of the residuals, while allowing a few residuals to be large. Furthermore, since the best PRISM line goes near point H, the corresponding residual for point H is fairly small. This demonstrates that examination of the residuals from a least squares fit provides no guarantee of finding all outliers. Indeed, outliers will generally not stand out as well in residual plots from least-squares fits (such as PRISM) as they will in residual plots from robust fits (such as RPRISM).

For house J43 (Figures 16-19), point C (Feb 16-Mar 18, 1979) is a very extreme outlier (Final Weight = 0.10). This forces the PRISM estimated value of τ down to 45.2°F. The RPRISM value of τ is higher, at 53.5°F. This higher value of τ allows points J, K, and D (which had few degree-days at τ = 45.2°F) to move to the right, making for a much better-defined line. This is the house with the highest CVR for NAC, 4.6. Note from the residual plots that point C is much further from the RPRISM line than it is from the PRISM line. This occurs since robust fitting methods do not strain to accommodate outliers at the expense of the bulk of the data. The resulting benefit for RPRISM is clear from the residual plots: the RPRISM residuals for all points except C and L are much smaller than the corresponding PRISM residuals.

For house J47 (Figures 20-23), point C (Feb 21-Mar 20, 1979) is a strong outlier (Final Weights of = 0.14), which in PRISM tends to exaggerate the heating slope, and lower R^2 (0.87 for PRISM, 0.96 for RPRISM). Indeed, the RPRISM slope is less than the PRISM slope by 5.3%. (Interestingly, the PRISM and RPRISM estimates of τ are only 1^{o}F different.) By allowing points C and L to have large residuals, RPRISM manages to decrease the remaining residuals. The RPRISM estimates are much better determined than the PRISM estimates. The CVR of NAC, for example, is 2.8.

To assess the stability of RPRISM in the face of extreme outliers, we performed an experiment on house J44. The results are summarized in Table 10. In the original data, point D (Mar 2-Apr 20, 1979) is a moderate outlier (Final Weight = 0.38), and the only outlier; the resulting PRISM and RPRISM estimates are very similar. The experiment consisted of artificially increasing the consumption for point D from 1940 kwh to 3000 kwh. This has the effect of increasing the PRISM estimate of τ from 59.4 to 66.2^{o}F, noticeably changing the estimated parameters, and markedly inflating the standard errors. By contrast, the RPRISM estimates and standard errors change hardly at all (for example, in the 6th decimal place of NAC). The weight for point D drops from 0.382 to 0.081, while all other weights remain at 1.0. The final comparison for J44 was with point D completely omitted. Not surprisingly, the resulting RPRISM and PRISM estimates are very close .

III.A.2. A Closer Look at the RPRISM Weights

The initial and final weights of the RPRISM runs on each house in this data set are listed in Table 11. (By "initial weights", we mean the weights computed after the first iteration of RPRISM, and thus the weights based on the residuals from PRISM least squares.) From this table, we see that the majority of all weights are 1.0. Furthermore, the initial and final weights for a given house often are not appreciably different, although there are exceptions. (See, for example, J27, J45, J47.) For three of the houses (J7, J50, and J57), the weights were all equal to 1.0. Note that as long as the initial weights are all 1.0, the same will be true of all subsequent weights. In this case, the final results will be identical to those given by PRISM. Indeed, the PRISM and RPRISM results do match for houses J7, J50 and J57, although there are small discrepancies due to rounding errors.

As we have mentioned, the final weights can be used for outlier identification: the smaller the weight, the more severe the outlier. To make this relationship more precise, it is useful to know the frequency with which weights of various sizes are observed. The second column of Table 12* gives the percentage of final weights (W) which are less than W, for various values of W. Thus, for example, 19.6% of the weights are less than 1.0, while only 1.5% are less than 0.3.

The observed final weights provide convincing evidence that least squares is <u>not</u> the optimal method for analyzing actual consumption data. This can be seen by comparing the observed weights with the weights that would be expected if the residuals from the robust fit had the normal ("bell-shaped curve") distribution.

We do this by using simulated data. Least-squares estimates are statistically optimal only if the error term ϵ_i in the model

$$Y_i = \alpha + \beta H_i(\tau) + \epsilon_i$$

has a Normal distribution. If, on the other hand, the error term (which we are approximating here by the residual from the robust fit) shows a higher proportion of outliers than admitted by the Normal distribution, then least squares is no longer optimal, in which case robust methods (such as RPRISM) often exhibit superior performance.

Column 3 of Table 12 summarizes the distribution of "weights" based on 200 simulated (computer generated) samples of Normal "residuals", each sample of size 12. Comparison of columns 2 and 3 shows some significant differences. For example, only 0.25% of the simulated weights were less than 0.3, while fully 1.5% of the observed weights were this small. Thus, extreme outliers (with weights < 0.3) occurred six times as often in the observed data as we would expect under the idealized assumptions which would have guaranteed the optimality of least squares.

III.A.3. A Comparison of Robust and Ordinary R^2

An objective of RPRISM was to develop a robust analogue of the R^2 statistic used in ordinary (least-squares) PRISM. In Figure 24,* we compare the robust R^2_w given by RPRISM with the ordinary R^2 of PRISM, where w corresponds with the final set of weights used in RPRISM.

Perhaps the most striking aspect of Figure 24 is that R^2_w is greater than R^2 for all houses. The median difference is 0.52%. The points which lie farthest from the $R^2_w = R^2$ line correspond to houses J47, J43, J63 and J23, for which the differences are 9.9, 7.5, 6.4 and 3.4%, respectively. In each case, the discrepancy can be attributed to the presence of one or more small final weights (corresponding to outliers). For houses J7, J50 and J57, R^2_w and R^2 are identical, since the final weights (as observed previously) were all 1.0 for these houses. Finally, since most of the points in Figure 24 lie reasonably close to (although always above) the $R^2_w = R^2$ line, we conclude that for non-anomalous data sets the robust and ordinary R^2 statistics behave similarly.

III.B. A Data Base of 50 Electrically Heated Houses with Central Air Conditioning

The Elec-AC data base consists of 50 houses located in New Jersey, which have electric space heating and central air conditioning. Our previous studies (Stram and Fels, 1986) showed that the air conditioning usage in many of these houses is evidently weather dependent, but in many others it is sporadic. (In all houses, weather-dependent heating consumption dominates.) The sporadic cooling creates difficulties with the heating-only PRISM model, since it contributes unexplained variability to the data in the form of outliers in the cooling months[1]. In particular, the sporadic cooling tends

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[1] Also under study is a heating-plus-cooling (HC) version of (ordinary) PRISM in which the cooling component as well as the heating component of consumption are modeled. [More precisely, the term $\beta_c C_i(\tau_c)$ is added to the

to inflate the estimate of the base level, α. Thus, it is hoped that if RPRISM is run on these houses, the sporadic cooling months will be largely ignored, and that the weather dependence of the bulk of the non-winter months will be accurately captured. Since RPRISM is an automated method, its use in this situation is preferable to a segmented approach in which outliers are first identified and removed and then PRISM is run. In addition, using RPRISM here seems clearly preferable to an arbitrary rejection rule such as: "throw away all consumption periods ending in July and August." It must be recognized, however, that if there is a long and consistent cooling period, then the cooling months will no longer be outliers, and in such cases RPRISM and PRISM will behave similarly.

Tables 13 and 14 give a summary of the PRISM and RPRISM results, respectively. Table 15* gives the mean and median values of the parameters, R^2, and coefficients of variation. Note that the RPRISM median value of α is 2.3% lower than the corresponding PRISM median value of α. With the exception of α, the PRISM and RPRISM average results are quite similar. Table 16 gives the percent differences for each house, and Figures 25-28 give the corresponding SAS Univariate summaries. Percent differences exceeding 10% occur for houses J2832, J2902, J2923, J3075, J3076, and J3112. The stemleaf diagram for the percent difference in α (Figure 25) shows a distribution which is clearly skewed towards low values: α from RPRISM is lower than the corresponding α from PRISM by as much as 13%, but higher by only as much as 2%. Thus, there is a clear tendency for α to decrease when RPRISM is applied, and this result is in agreement with intuition: if the sporadic cooling months are downweighted, α will go down. The skewness

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model in Eq. (1)]. Although the resulting five-parameter model has been a useful research tool, there are stability problems that complicate the interpretation of the individual parameter estimates. Whether robustifying the HC model would alleviate these problems warrants consideration.

described above is also apparent in the plots of the quartiles of the percent differences, in Figure 29,* and in Table 17,* which gives the means, medians, quartiles, and extremes of the percent differences. We believe that the lower α, from RPRISM, is a more reliable estimate of base-level consumption, insofar as α is intended to represent the temperature-independent component of consumption and thus should exclude a cooling contribution.

The CVRs are listed in Table 18, and the corresponding SAS Univariate results are given in Figures 30-33. Table 19* gives the mean, median, quartiles and extremes of the CVRs. The quartiles of the CVRs are plotted in Figure 34.* The average CVRs are all greater than one, but are not as large as they were for the Elec-HO houses.

A scatterplot of robust versus ordinary R^2 shown in Figure 35* illustrates that robust R^2 i.e., R^2_w (median = 0.982) is systematically higher than ordinary R^2 (median = 0.975). Similar to the Elec-HO sample, R^2_w is systematically higher than R^2 for all houses.

To see whether the changes that RPRISM does produce can be linked to sporadic cooling, we now examine in detail three interesting cases. These are houses J2832, J3075 and J3076.

For house J2832, the consumption vs. period plot (Figure 36) indicates relatively strong cooling in period H (Jul 26-Aug 23, 1979). This is borne out in the PRISM consumption vs. HDD and residuals vs. HDD plots (Figures 37, 38). The corresponding RPRISM plots (Figures 39, 40) show that point H is indeed a strong outlier (Final Weight = .023). Other outliers are D,C and I, with Final Weights of 0.28, 0.42 and 0.67, respectively. Note that RPRISM achieves a very good linear relationship among the non-outlying points. The α estimate goes from 74.1 to 64.5 (down by 13%), and R^2 goes from 0.83 to 0.93.

For house J3075, the consumption vs. period plot (Figure 41) shows no indication of summer cooling. The PRISM and RPRISM consumption vs. HDD plots (Figures 42, 43) show that the discrepancies can be attributed to the outliers B and C (Final Weights = 0.54, and 0.61 respectively). The fact that α increases a small amount, from 41.3 to 42.9 (by 4%), should not worry us, since there was no evidence of cooling for this house.

For house J3076, the consumption vs. period plot (Figure 44) suggests an overwhelming cooling load in the single period I (Aug 2-Aug 30, 1979). The PRISM consumption vs. HDD and residuals vs. HDD plots (Figures 45, 46) show that point I is indeed a gross outlier. The corresponding RPRISM plots (Figures 47, 48) show that the downweighting of point I (Final Weight = 0.16) allows for a more reasonable intercept, and for smaller residuals in the non-outlying points. The α estimate decreases from 98.9 to 86.6 (by 12%), and R^2 increases from 0.82 to 0.95.

It is clear from these three houses that, in comparison with heating, the contribution to consumption from cooling in a climate such as New Jersey's can be weak and erratic. One motivation for this study was to explore whether a robust version of the heating-only PRISM model could reduce the interference of cooling on the heating estimates. The results for the Elec-AC sample, on average for the 50 houses and individually for these three cases, suggest a high degree of success. More work is needed to compare the RPRISM estimates with those resulting from our experimental heating-plus-cooling model. Ultimately a comparison with submetered heating and cooling consumption in houses heated and cooled by electricity would be desirable.

IV. THE MERITS OF WEIGHTED PRISM WHEN THE PERIOD LENGTHS DIFFER

The basic PRISM model assumes that all data values have the same variance, and hence are equally reliable. This is a reasonable assumption if the period lengths are all nearly identical. If the period lengths are radically different, however, then the most sensible assumption is that the variance of a given data point is inversely proportional to the length of the corresponding period. Thus, if the period lengths differ radically, as can be the case with oil-heated homes, the constant-variance assumption is inappropriate. In addition, even if we put aside robustness considerations, ordinary least-squares regression is no longer the optimal method of estimating the parameters. If we use PRISM in this case, the estimates will not be as well determined as they could be. Further, the PRISM standard errors will be incorrect since the theory used in PRISM relies on the equal variance assumption. It can be shown (see Appendix AV) that the optimal estimation technique here is weighted least squares with weights proportional to the period lengths. Thus, if we use weighted PRISM with these weights, then we can get better estimates and correct standard errors.

IV.A. A Data Base of 69 Oil-Heated houses: Ordinary vs. Weighted PRISM

We now present a comparison of ordinary and weighted PRISM (PRISM and OUTWTS, respectively) on a data base (OIL) of 69 oil-heated houses in New Jersey. Thirty-nine of these houses have an oil-fired hot water heater, and therefore have a theoretically positive oil base level. These houses are designated as "HW", vs. "H" for the remaining houses that use oil for heating only. (The application of regular PRISM to this data base was studied previously, in Fels et al., 1986.)

We use the name WPRISM to refer generally to weighted PRISM, and the name OUTWTS to refer specifically to WPRISM with weights proportional to the period lengths. This terminology stresses that here WPRISM is being run on fixed outside weights that do not depend directly on consumption. Note that RPRISM, by contrast, consists of WPRISM run iteratively on robust or inside weights computed from the residuals. (See the schematics for OUTWTS and RPRISM, Figures 49* and 1,* respectively.) In the present study, we are concentrating on the effects of the outside weighting, and therefore do not use the robust option. In principle, outside weighting (OUTWTS) and robust PRISM (RPRISM) can be used simultaneously; one such case is presented later in this section.

Tables 20 and 21 give complete summaries of the PRISM and OUTWTS results, respectively. The water-heater indicator (HW,H) is included. Of the 39 houses with HW, negative α occurs consistently between PRISM and OUTWTS, i.e., the set of eight HW houses for which PRISM gave negative α is identical to the set for which OUTWTS gave negative α. (The HW indicator is a homeowner response and is not always an accurate indicator of the type of water heater. As indicated previously, several of these cases were miscoded.)

Table 22* gives the mean and median values of R^2, the parameters, and their coefficients of variation (CV). For CV(α), the HW houses with positive α and finite se(α) (i.e., finite se(τ)[1]) are treated separately: there were 31 such houses for PRISM, 30 for OUTWTS. In our earlier study of the same data base, the PRISM parameters were generally less well determined for oil-heated houses than for electrically or gas-heated houses (Fels et al., 1986). A comparison of the CVs in Table 22 with those in either Table 4

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[1] When se(τ) is infinite, as it will be if "best TAU" is determined to be the maximum observed temperature in the time period used for estimation, then the corresponding se(α) is assumed to be infinite as well. In this data set, there were 3 such cases with OUTWTS and 5 such cases with PRISM where this occurred. All in all, there were 8 cases with large or infinite se(τ)(i.e., se(τ) > 20), and they were the same set of 8 with both PRISM and OUTWTS.

or 15 is consistent with our earlier observations. Clearly, part of the discrepancy between oil and the other results is due to noneven period lengths in the oil data: weighted PRISM reduces the CV of NAC for the oil data base from a median of 6.8% to a median of 4.1%, thus bringing it more in line with the 3.0% CV typically seen for gas- and electrically heated houses.

Table 23 gives the by-house percent differences for the estimates, and the corresponding SAS Univariate outputs are given in Figures 50-53. The mean, median, quartiles and extremes are given in Table 24.* The quartiles of the percent differences are plotted in Figure 54.* Some of the percent differences in α are quite large because α, especially for houses without oil water heating, may be very close to zero. Still, the percent differences in the other parameters as well seem much greater than they were for the earlier comparisons of PRISM and RPRISM.

The CVRs are given in Table 25, with the corresponding SAS univariate output in Figures 55-58. The Univariate summaries for CVR(α) and CVR (heating part) are based on the 64 homes with finite se(τ). Table 26* gives the means, medians, quartiles and extremes of the CVRs. The quartiles of the CVRs are plotted in Figure 59.* For CVR(α), only the 30 HW houses with positive α and finite se(α) for both PRISM and OUTWTS are included. Again, some of the CVRs are very large, and the ranges are much larger than they were in the comparison of PRISM and RPRISM for electrically heated houses. The median value for CVR(α) was 1.46, and the mean was 3.76, vs. a median of 1.08 for both electricity data sets. Thus, α was substantially better determined by the OUTWTS method. A plausible explanation for this result is that many of the long periods fall in the summer (in which case there might be only one "summer" data point). Hence if these points are given large weights, as they are by the OUTWTS option, then the base level will be better determined. In addition, NAC is considerably improved by the weighting: the median CVR(NAC) from OUTWTS vs. PRISM applied to oil-heated houses is 1.61, vs. the comparable medians of 1.11 and 1.08 from RPRISM vs. PRISM applied to electricity data sets Elec-HO and Elec-AC.

A scatterplot of R^2 (Figure 60)* shows that the R^2 values are consistently higher for OUTWTS than for PRISM. The median value was 0.982 for OUTWTS, up from 0.971 for PRISM. The large average values of CVR(α) and CVR(NAC), together with the increase in R^2, indicate that the weighting procedure tried here is indeed reasonable.

Two houses which showed great differences between the PRISM and OUTWTS results were P76263 and P57490. We now examine these houses in detail.

For P76263, an HW house, the consumption versus period plot (Figure 61) shows a very erratic pattern, with consumption varying wildly from one period to the next. The PRISM consumption vs. HDD plot (Figure 62) shows a poor linear fit, with a particularly strong outlier at point B (Nov 12-Nov 12, 1979 and an unreasonably low value of τ=37.0°F. Interestingly, the period length corresponding to point B is just one day -- perhaps a data error, but an interesting test case for our purposes. (The second shortest period was F (Jan 11-Jan 24, 1980), with 14 days.) The OUTWTS consumption vs. HDD plot (Figure 63) shows a line which better fits the lower cluster of points (A,C,F,G,H,I). The change in τ was substantial, from 37.0 to 75.0°F, while R^2 increased from 0.20 to 0.65. The α estimate is cut in half, from 8.98 to 4.43 gal/day. Unfortunately, the standard error of α goes from 0.97 to indeterminable [se(τ) = ∞, but this seems an acceptable price to pay for the improved model fit.

For house P57490, another HW house, the consumption vs. period plot is given in Figure 64. The summer points B (Apr 10-Aug 12, 1979) and J (Apr 23-Aug 27, 1980) have the longest periods. The PRISM consumption vs. HDD plot (Figure 65) shows a fairly good fit, with R^2 = .965. The corresponding

OUTWTS plot (Figure 66) is similar, except for a shift in the horizontal axis due to the change in τ from 76.5 to 72.0°F. The added weight placed on points B and J pulls the line closer to these points, and thereby brings α up from 0.02 to 0.45. The standard error of α decreases from 1.54 to 0.57, and R^2 increases from 0.965 to 0.973. Note that point I is a moderate outlier which is not downweighted by the OUTWTS method since it does not correspond to an unusually short period. Running the program with both the ROBUST and OUTWTS options changes α to 0.35, and increases R^2 to 0.988.

On the basis of the study described in this section, we can conclude that OUTWTS improves the model fit, as well as the determination of the parameters, for oil-heated houses having unequal period lengths. In particular, the determination of α (and therefore of NAC) is often improved due to the increased weights given to summer points. This phenomenon was illustrated in our study of house P57490. In addition, OUTWTS can improve the model fit by downweighting points with unusually short period lengths, as shown in our study of house P76263.

V. CONCLUSION

We have developed a robust version of PRISM as well as an automatic adjustment for unequal period lengths, and we have studied the performance of these new methods on over 150 houses. The new methods retain all of the features of ordinary PRISM (e.g., variable reference temperature, standard errors for the parameter estimates, R^2 statistic to summarize adequacy of model fit). The execution of the new computer algorithm is identical to that of PRISM, with the requirement of additional input commands.

By automatically finding and downweighting outliers, robust PRISM (RPRISM) can often improve the model fit as well as the determination of the parameters. Outliers can occur even in houses with no apparent problems, can be difficult and time-consuming to detect by hand, and can severely deteriorate the corresponding PRISM results. Hence, the development of an objective and automatic method to deal with these problems is worthwhile. When there are no outliers, RPRISM and PRISM will give identical results.

For a set of 50 electrically heated houses without air conditioning, the median CV of NAC was 2.4% for RPRISM, versus 3.0% for PRISM. The RPRISM values of R^2 were systematically higher than those for PRISM: the median R^2 value was 0.981 for RPRISM, vs. 0.972 for PRISM. PRISM and RPRISM often give reasonably similar results, thereby reinforcing our confidence in PRISM as a useful tool. A study of individual problem houses, however, shows that RPRISM can make noticeable improvements.

For a set of 50 electrically heated houses with central air conditioning the overall gains of RPRISM were not as strong. The median CV of NAC was 2.7% for RPRISM, versus 3.0% for PRISM. Still, RPRISM was able to make strong improvements in some houses exhibiting sporadic cooling and other isolated outlier problems. In houses with sporadic cooling, PRISM generally overestimates the base level α. RPRISM, by automatically downweighting the outlying summer points, typically decreased (and thus improved) α. The median R^2 value was 0.982 for RPRISM, vs. 0.975 for PRISM; robustness systematically improved R^2 for this data set as well.

The adjustment for unequal period lengths (OUTWTS) can be justified on theoretical grounds, and provides a straightforward application of weighted PRISM. In a study of 69 oil-heated homes (many of which had periods of radically different lengths), OUTWTS was able to improve the model fit and accuracy (often dramatically so), by giving increased weight to the summer points and decreased weight to points with unusually short period lengths. The increased weighting of summer points improved the determination of α and hence of NAC. The median CV of NAC was 4.1% for OUTWTS, vs. 6.8% for PRISM.

R^2 was systematically improved by OUTWTS. The median R^2 value was 0.982 for OUTWTS, vs. 0.971 for PRISM.

The sample applications illustrate the improvements in reliability of PRISM estimates that can occur from a more robust treatment (internal weighting) of outliers in consumption data, and also from external weighting of different consumption period lengths. The improvements can be dramatic, particularly in cases with one or more extreme outliers (or extremely uneven period lengths). Yet, in the three samples presented here, and in samples we have analyzed elsewhere, such cases are the exception rather than the rule.

An important question for evaluators of conservation programs is whether ordinary PRISM is adequate for the determination of average savings. We believe that it generally is, particularly if robust statistics (e.g., median rather than mean values) are used to calculate the averages; improvement of a few anomalous cases in a sample of hundreds should not greatly affect the median. There are many examples of evaluations in which PRISM has produced highly reliable, and statistically significant, estimates of average savings (see, for example, Dutt et al., 1986). If, on the other hand, the concern is for as much reliability as possible for each individual house being analyzed, use of robust PRISM may substantially increase the quality of the study.

One such study, recently conducted by Princeton, was a pilot test of a Home Energy Report in which consumers were offered weather-adjusted estimates of their own consumption over the last two years (Layne et al., 1988). It was important to offer the estimates only if they were reliable, and to maximize the number of participants that received the estimates. For this reason, robust PRISM was used throughout the study. Although most of the participants (53 out of 64) would have received reliable weather adjustments with either PRISM or RPRISM, use of RPRISM made it possible to increase the number of participants receiving reliable information (from 53 to 60;

Reynolds and Fels, 1988). Figures 67* and 68* respectively show a plot of CV(NAC) and NAC for Robust vs. Ordinary PRISM. Whereas on average CV(NAC) improves only a little (for example, median CV(NAC) decreases from 0.033 with PRISM to 0.029 with RPRISM for the first year of data), a dramatic improvement in CV(NAC) for a few individual cases is evident. Nevertheless, NAC is not greatly changed by the robust version, on average or even at the level of individual cases. This is consistent with the findings of the Elec-HO and Elec-AC samples studied here (Figures 4 and 27). Robust PRISM seems to have a greater effect on the accuracy of the estimates than on the estimates -- a feature that could be particularly useful when individual-house or individual-building accuracy is of paramount importance.

The PRISM software currently being distributed (version 4.0, dated October 1986) contains regular PRISM, with a cooling-only as well as a heating-only option. Since the vast majority of applications seek a high level of reliability in <u>average</u> savings, this version seems to be meeting the needs of the more than 150 members of the PRISM Users Network. We hope at some time in the near future to be able to add robust and weighted PRISM, as options to the software package.

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AI: Description of RPRISM

The key to robustifying PRISM lies in modifying the original least squares method to allow for *weighted least squares* . Thus, if the heating degree days per day at base τ and the consumption are

$$(x_i(\tau), y_i)$$

for period i , $i = 1, \ldots, n$, then we want to find $\hat{\alpha}, \hat{\beta}$, and $\hat{\tau}$ to minimize

$$\sum_{i=1}^{n} w_i (y_i - \hat{y}_i)^2 \quad , \tag{1}$$

where

$$\hat{y}_i = \hat{\alpha} + \hat{\beta} x_i(\tau) \quad ,$$

and the w_i are a sequence of fixed nonnegative weights. Obviously, by setting all the weights to 1, we get regular PRISM. But if we arrange things so that "outliers" get small weights while the rest of the points get weights of 1, then the estimates from the weighted least squares version of PRISM will be insensitive to the outliers, and hence will be more robust than the estimates obtained from regular PRISM. To obtain the solution $\hat{\alpha}, \hat{\beta}, \hat{\tau}$ to the weighted least squares problem (1), we need to change the basic Newton's Method algorithm used by PRISM. We will give details of these modifications in Section AIV, but first we describe the complete RPRISM algorithm, of which weighted least squares is just a part.

RPRISM is an adaptation of the Huber M-Estimate, using iteratively reweighted least squares (IRLS) as the numerical method to obtain the estimate. In M-Estimation, the idea is to find estimates to minimize

$$\sum_{i=1}^{n} \rho \left[\frac{y_i - \hat{y}_i}{\hat{\sigma}} \right] \tag{2}$$

where ρ is a fixed function, and $\hat{\sigma}$ is a robust estimate of scale. If we put $\rho(r) = r^2$, we get least squares. If we put $\rho(r) = |r|$, we get least absolute deviations (the analog of the one-dimensional median). Huber proposes a compromise between these:

$$\rho(r) = \begin{cases} r^2/2 & |r| \le H \\ H|r| - H^2/2 & |r| > H \end{cases} \quad H = 1.345 \quad .$$

The resulting estimates can be shown to be robust (even "optimally" robust, under certain assumptions). Unfortunately, the solutions to (2) cannot, in general, be written in closed form, so we must resort to iterative numerical techniques. We have chosen to use IRLS (which will be described presently) as that numerical technique. It can be shown that the solution to IRLS converges to the solution to (2) as the number of iterations becomes large.

AII: Specification of the IRLS Algorithm For Computing RPRISM Estimates

The IRLS algorithm proceeds iteratively, and involves solving a weighted least squares problem (1) at each stage. The weights are determined by the residuals from the previous fit, through a simple formula. We will describe IRLS in the context of PRISM:

1) Start with an ordinary least squares fit. Thus, find the solutions $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ to (1) with $w_i = 1$, $i = 1, \cdots, n$.

2) Using the current estimate $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$, compute the residuals $r_i = y_i - \hat{y}_i = y_i - (\hat{\alpha} + \hat{\beta} x_i(\tau))$.

3) To obtain a robust estimate of scale, compute the median absolute deviation (from the median) of the residuals:

$$MAD = \underset{i}{med} \left| r_i - \underset{j}{med}\{r_j\} \right| \quad .$$

The final scale estimate is then

$$\hat{\sigma} = 1.48\, MAD \quad .$$

4) Obtain new weights from the residuals $\{r_i\}$ and the scale estimate $\hat{\sigma}$ by the formula

$$w_i = \begin{cases} 1 & |r'_i| \le H \\ H/|r'_i| & |r'_i| > H \end{cases} ,$$

where $\{r'_i\}$ are the standardized residuals:

$$r'_i = \frac{r_i}{\hat{\sigma}} \quad .$$

5) Get new estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ by weighted least squares (1) (see Section AIV), using the weights w_i from step 4).

6) If the current estimates are "sufficiently close" to the previous estimates, stop. Otherwise, go to step 2).

Of course, we use the estimates obtained in the final iteration. The weights that were used to obtain these estimates (i.e., the "final weights") are useful for outlier detection: the "outliers" are the points with the small weights. The smaller the weight, the more extreme the outlier.

AIII: Standard Errors for the RPRISM Estimates

Given the robust estimates of α, β, τ, NAC and the heating part of NAC, we can obtain approximate standard errors for these estimates as follows:

1): The variance covariance matrix for

$$\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \\ \hat{\tau} \end{bmatrix}$$

is

$$\begin{bmatrix} Var\,\hat{\alpha} & Cov(\hat{\alpha},\hat{\beta}) & Cov(\hat{\alpha},\hat{\tau}) \\ Cov(\hat{\alpha},\hat{\beta}) & Var\,\hat{\beta} & Cov(\hat{\beta},\hat{\tau}) \\ Cov(\hat{\alpha},\hat{\tau}) & Cov(\hat{\beta},\hat{\tau}) & Var\,\hat{\tau} \end{bmatrix},$$

and it is estimated by

$$M = \frac{n}{n-3}\hat{\sigma}^2 (\dot{\eta}^T \dot{\eta})^{-1} \frac{\frac{1}{n}\sum \Psi^2(r'_i)}{\left[\frac{1}{n}\sum \Psi'(r'_i)\right]^2}.$$

The various quantities in this equation are defined below.

A) $\hat{\sigma}$ is the current robust scale estimate obtained from the IRLS routine.

B)

$$\dot{\eta} = \begin{bmatrix} 1 & x_1(\hat{\tau}) & \hat{\beta}F_1(\hat{\tau}) \\ 1 & x_2(\hat{\tau}) & \hat{\beta}F_2(\hat{\tau}) \\ : & : & : \\ 1 & x_n(\hat{\tau}) & \hat{\beta}F_n(\hat{\tau}) \end{bmatrix}$$

where

$$F_i(\hat{\tau}) = \frac{d}{d\tau}x_i(\tau)\,|_{\tau=\hat{\tau}}\ .$$

Note that $\dot{\eta}$ is the same matrix as defined on Page 74 of Goldberg (1982), Equation (4.13).

C)

$$r'_i = \frac{r_i}{\hat{\sigma}} \quad \text{where } r_i = y_i - (\hat{\alpha} + \hat{\beta} x_i(\hat{\tau})) \ .$$

D)

$$\Psi^2(r) = \begin{cases} r^2 & |r| \le H \\ H^2 & |r| > H \end{cases} ,$$

$$\Psi'(r) = \begin{cases} 1 & |r| \le H \\ 0 & |r| > H \end{cases} .$$

2): The standard errors of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ are

$$SE\ \hat{\alpha} = \sqrt{M(1,1)} \ , \ SE\ \hat{\beta} = \sqrt{M(2,2)} \ , \ SE\ \hat{\tau} = \sqrt{M(3,3)} \ .$$

3) The standard error of $NAC = 365.25(\hat{\alpha} + \hat{\beta} + H_o(\hat{\tau}))$ is:

$$\begin{aligned} SE\ \hat{\Gamma} = 365.25(&Var\ \hat{\alpha} + (H_o(\hat{\tau}))^2 Var\ \hat{\beta} + (\hat{\beta} F_o(\hat{\tau}))^2 Var\ \hat{\tau} \\ &+ 2H_o(\hat{\tau})\, Cov(\hat{\alpha}, \hat{\beta}) + 2H_o(\hat{\tau})\, \hat{\beta} F_o(\hat{\tau})\, Cov(\hat{\beta}, \hat{\tau}) \\ &+ 2\hat{\beta} F_o(\hat{\tau})\, Cov(\hat{\alpha}, \hat{\tau}))^{1/2} \ , \end{aligned}$$

where the covariances are found as the appropriate entries of the matrix M found in step 1. Note that $H_o(\hat{\tau})$ is the normalized heating degree days *per day* at base $\hat{\tau}$.

4) The standard errors for the heating part of NAC ($= 365.25\, \hat{\beta} H_o(\hat{\tau})$) is given by

$$\begin{aligned} SE\ \text{Heating} = 365.25(&(H_o(\hat{\tau}))^2 Var\, \hat{\beta} + (\hat{\beta} F_o(\hat{\tau}))^2 Var\, \hat{\tau} \\ &+ 2\hat{\beta} F_o(\hat{\tau}) H_o(\hat{\tau})\, Cov(\hat{\beta}, \hat{\tau}))^{1/2} \ . \end{aligned}$$

AIV: Implementation of WPRISM: Hilbert Space Approach

The WPRISM (weighted PRISM) problem is: find $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ to minimize

$$\sum_{i=1}^{n} w_i (y_i - (\alpha + \beta H_i(\tau)))^2 \ ,$$

where the w_i are fixed weights, $0 < w_i \le 1$, $i = 1, \ldots, n$. The solutions to WPRISM are useful for at least two purposes: first, as a building block in the fully robust RPRISM algorithm; second, as a stand-alone method in situations where the period lengths are radically different.

Here, we will show that the solutions to WPRISM can be obtained from the ordinary PRISM algorithm if certain sums are changed to weighted sums. Our development hinges on a simple observation about vector space norms and inner products, which we haven't seen exploited elsewhere. Besides giving a simple solution to WPRISM, our idea leads to a new and previously unexplored generalization of the R^2 statistic having interesting geometrical and robustness properties.

Before presenting our idea, we will give some elementary background on norms and inner products of vectors, and on linear regression. If x and y are n-dimensional vectors $x=(x_1,\ldots,x_n)^T$, $y=(y_1,\ldots,y_n)^T$, then

$$(x,y)=\sum_{i=1}^{n} x_i y_i$$

is called the Euclidean inner product of x and y, and

$$||x|| = \sqrt{\sum x_i^2} = \sqrt{(x,x)}$$

is called the Euclidean norm of x. If $(x,y)=0$, then x and y are said to be *orthogonal*. If, in addition, $||x|| = ||y|| = 1$, then x and y are said to be orthonormal.

The simple linear regression model can be written as

$$y_i = \alpha + \beta x_i + \varepsilon_i \quad i=1,\ldots,n \quad ,$$

or

$$y = \alpha 1 + \beta x + \varepsilon \quad ,$$

where $1=(1,\ldots,1)^T$ and $x=(x_1,\ldots,x_n)^T$. The least squares estimates $\hat{\alpha}$, $\hat{\beta}$ minimize $||y-\hat{y}||^2$, where $\hat{y}=\alpha 1 + \beta x$, as a function of α, β. As shown in Theorem 1 (which relies on vector space concepts), the solutions are

$$\hat{\alpha} = \bar{y} - \frac{SXY}{SXX}\bar{x} \quad , \quad \hat{\beta} = \frac{SXY}{SXX} \quad ,$$

where

$$SXX = ||x-\bar{x}||^2 \quad , \quad SYY = ||y-\bar{y}||^2 \quad , \quad SXY = (x-\bar{x}, y-\bar{y}) \quad .$$

Now, the weighted least squares estimates, by definition, minimize

$$\sum_{i=1}^{n} w_i (y_i - \hat{y})^2 \quad .$$

Interestingly, the weighted least squares solutions can be expressed in formulas which have exactly the same form as those given above, if we replace the Euclidean norm $||x||$ and Euclidean inner product (x, y) by the *weighted* norm

$$||x||_w = \sqrt{\sum w_i x_i^2}$$

and the *weighted* inner product

$$(x, y)_w = \sum_{i=1}^{n} w_i x_i y_i \quad .$$

The proof follows simply from Theorem 1 and from the fact that the weighted least squares estimates minimize $||y - \hat{y}||_w^2$. For more details, see Theorem 2. Thus, if we define

$$S_w = \sum w_i \quad , \quad \bar{x}_w = \frac{\sum w_i x_i}{S_w} \quad , \quad \bar{y}_w = \frac{\sum w_i y_i}{S_w} \quad ,$$

$$SXX = ||x - \bar{x}_w||_w^2 = \sum_{i=1}^{n} w_i (x_i - \bar{x}_w)^2 \quad ,$$

$$SYY = ||y - \bar{y}_w||_w^2 = \sum_{i=1}^{n} w_i (y_i - \bar{y}_w)^2 \quad ,$$

$$SXY = (x - \bar{x}_w, y - \bar{y}_w) = \sum_{i=1}^{n} w_i (x_i - \bar{x}_w)(y_i - \bar{y}_w) \quad ,$$

then the weighted least squares estimates are

$$\hat{\alpha} = \bar{y}_w - \frac{SXY}{SXX}\bar{x}_w \quad , \quad \hat{\beta} = \frac{SXY}{SXX} \quad .$$

Note that these formulas reduce to the unweighted case if all w_i are set to 1.

We now turn to the PRISM model,

$$y = \alpha 1 + \beta H(\tau) + \varepsilon \quad .$$

We will show that the ordinary PRISM estimates of α, β, τ are in fact the least squares estimates. If τ is fixed, $\hat{\alpha}$, $\hat{\beta}$ are the corresponding least squares estimates of α, β, and $\hat{y} = \hat{\alpha} 1 + \hat{\beta} H(\tau)$, then the R^2 statistic is defined as

$$R^2(\tau) = 1 - \frac{||y - \hat{y}(\tau)||^2}{||y - \bar{y}||^2} = 1 - \frac{RSS(\tau)}{||y - \bar{y}||^2} \quad . \tag{1}$$

By Theorem 3 (with all weights set to 1), we can also write $R^2(\tau)$ as

$$R^2(\tau) = \frac{||\hat{y} - \bar{y}||^2}{||y - \bar{y}||^2} \quad . \tag{2}$$

Thus, $R^2(\tau)$ is the proportion of the total variability in y (i.e., $||y-\bar{y}||^2$) which can be explained by the least squares fit $\hat{y}(\tau)$. $R^2(\tau)$ is also the square of the correlation coefficient between y and $H(\tau)$, as shown in Theorem 3. As defined in most CEES reports, and as implemented in the original PRISM program, the PRISM estimate of τ is the value $\hat{\tau}$ which maximizes $R^2(\tau)$. But since $||y-\bar{y}||^2$ does not depend on τ, (1) implies that $\hat{\tau}$ can equivalently be characterized as the value which minimizes $RSS(\tau)$. Now, it follows easily that the PRISM estimates are the least squares estimates, i.e., that they minimize $||y-(\alpha 1+\beta H(\tau))||^2$ as a function of all three parameters α, β, τ. To see this, note that the minimizer $\hat{\tau}$ of $RSS(\tau)$ and the corresponding least squares estimates $\hat{\alpha}$ and $\hat{\beta}$ give the best least squares fit which can be obtained by holding τ at some fixed value. If there were some other numbers α^*, β^*, τ^* which gave a better fit than $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$, then the least squares estimates of α and β corresponding to τ^* would have to also give a better fit than $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$, and this possibility is ruled out by the discussion given above.

Thus for a given value of τ, the corresponding least squares estimates $\hat{\alpha}$, $\hat{\beta}$ are given by

$$\hat{\alpha}=\bar{y}-\frac{SHY}{SHH}\bar{H} \quad , \quad \hat{\beta}=\frac{SHY}{SHH}$$

where

$$SHH=||H(\tau)-\bar{H}(\tau)||^2 \quad , \quad SYY=||y-\bar{y}||^2 \quad , \quad SHY=(H-\bar{H}\,,y-\bar{y}) \quad .$$

Furthermore,

$$\hat{Y}=\hat{\alpha}1+\hat{\beta}H(\tau) \quad , \quad RSS(\tau)=||y-\hat{y}||^2 \quad ,$$

$$R^2(\tau)=1-\frac{RSS(\tau)}{||y-\bar{y}||^2}=\frac{SHY^2}{SHH\;SYY}=\hat{\beta}^2\frac{SHH}{SYY} \quad .$$

$\hat{\tau}$ is determined by maximizing $R^2(\tau)$, or, equivalently, by minimizing $RSS(\tau)$. In PRISM, this maximization is accomplished through a combination of Newton's Method and a grid search.

We will now show that the WPRISM estimates can be obtained from the original PRISM algorithm, provided that $\bar{H}$, $\bar{y}$, $||\;||$, $(,)$ are changed to $\bar{H}_w$, $\bar{y}_w$, $||\;||_w$, $(,)_w$. Thus, if we make these simple changes, we can use the existing PRISM software to solve the WPRISM problem. We will also discuss some of the properties of R_w^2, a new version of the R^2 statistic which is appropriate for WPRISM. First, it follows from Theorem 2 that for τ fixed, the weighted least squares estimates of α and β are given by

$$\hat{\alpha}=\bar{y}_w-\frac{SHY}{SHH}\bar{H}_w \quad , \quad \hat{\beta}=\frac{SHY}{SHH} \quad ,$$

where

$$SHH = ||H(\tau)-\bar{H}_w(\tau)||_w^2 \quad , \quad SYY = ||y-\bar{y}_w||_w^2 \quad , \quad SHY = (H-\bar{H}_w \, , y-\bar{y}_w)_w \quad .$$

Define

$$\hat{y} = \hat{\alpha}1 + \hat{\beta}H(\tau) \quad , \quad RSS(\tau) = ||y-\hat{y}||_w^2 = \sum w_i(y_i-\hat{y}_i)^2 \quad ,$$

$$R_w^2(\tau) = 1 - \frac{RSS(\tau)}{||y-\bar{y}_w||^2} \quad . \tag{3}$$

R_w^2 is a statistic which has not, so far as we know, been previously studied. Since by Theorem 3,

$$R_w^2 = \frac{||\hat{y}-\bar{y}_w||_w^2}{||y-\bar{y}_w||_w^2} \quad ,$$

we can think of R_w^2 as measuring the proportion of the total "variability" in y (i.e., $||y-\bar{y}_w||_w^2$) which can be explained by the weighted least squares fit. Here, variability is measured in terms of the w-norm. As shown in Theorem 3, $R_w^2(\tau)$ is also the square of the correlation coefficient between y and $H(\tau)$, with respect to the w-norm and w-inner product. This guarantees that $0 \le R_w^2(\tau) \le 1$, and allows us to write $R_w^2(\tau)$ as

$$R_w^2(\tau) = \frac{SHY^2}{SHH\;SYY} = \hat{\beta}^2 \frac{SHH}{SYY} \quad ,$$

in exact analogy to the unweighted case. By virtue of (3), the WPRISM estimate $\hat{\tau}$ can be found by maximizing $R_w^2(\tau)$. The final WPRISM estimates are then the weighted least squares estimates $\hat{\alpha}$, $\hat{\beta}$ corresponding to $\hat{\tau}$, along with $\hat{\tau}$ itself.

So far, we have shown that for fixed τ, PRISM can be changed to WPRISM by simply changing the appropriate sums, norms and inner products to weighted sums, weighted norms, and weighted inner products. We have also shown that the WPRISM estimate of τ can be obtained by maximizing $R_w^2(\tau)$, in exact analogy to the ordinary PRISM case. To prove that the above changes will transform the full PRISM algorithm (in which τ is not necessarily fixed) into the full WPRISM algorithm, we still need to show that if the above changes are made, the formulas used in the PRISM Newton's Method will yield a valid Newton's Method for WPRISM. The proof is given in Theorem 4, which closely parallels the development for PRISM given in Goldberg (1982), pp. 237-239.

We now give Theorems 1-4.

Theorem 1: For simple linear regression, the least squares solution $\hat{\alpha}$, $\hat{\beta}$ which minimizes $||y-(\alpha 1+\beta x)||^2$ is

$$\hat{\beta}=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}=\frac{SXY}{SXX} \quad , \quad \hat{\alpha}=\bar{y}-\hat{\beta}\bar{x} \ .$$

Proof: Define

$$v=\frac{1}{\sqrt{n}}1 \ , \ SXX=||x-\bar{x}||^2 \ , \ SYY=||y-\bar{y}||^2 \ , \ SXY=(x-\bar{x},y-\bar{y}) \ , \ u=\frac{x-\bar{x}}{\sqrt{SXX}} \ .$$

Then u and v are orthonormal and y can be expressed uniquely as

$$y=(y,v)v+(y,u)u+\sum_{j=3}^{n}(y,v_j)v_j \ ,$$

where v_j $j=3,\ldots,n$ are orthonormal vectors each of which is also orthonormal to v and u . Note that any linear combination of 1 and x can be uniquely expressed as a linear combination of v and u (and *vice versa*). It can be shown that the minimizer $\hat{y}=\hat{\alpha}1+\hat{\beta}x$ of $||y-(\alpha 1+\beta x)||^2$ is characterized by $(y-\hat{y},z)=0$ for any linear combination z of 1 and x (or equivalently, for any linear combination z of v and u). We will now prove that the minimizer $\hat{y}$ is given by

$$\hat{y}=(y,v)v+(y,u)u \ .$$

To prove it, we must show that $(y-\hat{y},u)=0$ and $(y-\hat{y},v)=0$. But these relations follow easily since

$$(y-\hat{y},u)=(\sum_{j=3}^{n}(y,v_j)v_j,u)=\sum_{j=3}^{n}(y,v_j)(v_j,u)=0$$

and

$$(y-\hat{y},v)=(\sum_{j=3}^{n}(y,v_j)v_j,v)=\sum_{j=3}^{n}(y,v_j)(v_j,v)=0 \ .$$

We have now proved that $\hat{y}=(y,v)v+(y,u)u$. Since $(y,v)v=\bar{y}$ and since

$$(y-\bar{y},u)=(y,u)-(\bar{y},u)=(y,u)-(y,v)(v,u)=(y,u) \ ,$$

we have

$$\hat{y}=\bar{y}+(y-\bar{y},u)u=\bar{y}+\frac{SXY}{\sqrt{SXX}}\frac{x-\bar{x}}{\sqrt{SXX}}$$
$$=[\bar{y}-\frac{SXY}{SXX}\bar{x}]+\frac{SXY}{SXX}x \ .$$

Thus,

$$\hat{\beta} = \frac{SXY}{SXX} \quad \text{and} \quad \hat{\alpha} = \bar{y} - \frac{SXY}{SXX}\bar{x} \ .$$

Theorem 2: For simple linear regression, the weighted least squares solution $\hat{\alpha}$, $\hat{\beta}$ which minimizes $||y-(\alpha 1+\beta x)||_w^2$ is

$$\hat{\beta} = \frac{\sum w_i(x_i - \bar{x}_w)(y_i - \bar{y}_w)}{\sum (x_i - \bar{x}_w)^2} = \frac{SXY}{SXX} \ , \quad \hat{\alpha} = \bar{y}_w - \hat{\beta}\bar{x}_w \ ,$$

where

$$\bar{x}_w = \frac{1}{S_w}\sum w_i x_i \ , \quad \bar{y}_w = \frac{1}{S_w}\sum w_i y_i \ , \quad S_w = \sum w_i \ .$$

Proof: In the proof of Theorem 1, change all occurrences of $||\ ||$ to $||\ ||_w$ and $(,)$ to $(,)_w$. Change $\bar{x}$ to $\bar{x}_w$ and $\bar{y}$ to $\bar{y}_w$. Finally, change v to

$$v = \frac{1}{\sqrt{S_w}} 1 \ .$$

All steps of the proof of Theorem 1 will still be correct if these changes are made. Thus the conclusion of Theorem 1, with the indicated changes, is correct as well.

Theorem 3: For simple weighted linear regression, if R_w^2 is defined by

$$R_w^2 = 1 - \frac{RSS}{SYY} \ ,$$

then the following relations hold:

$$R_w^2 = \frac{||\hat{y} - \bar{y}_w||_w^2}{||y - \bar{y}_w||_w^2} \ , \quad RSS = SYY - \frac{SXY^2}{SXX} \ , \quad R_w^2 = \frac{SXY^2}{SXX\ SYY} \ .$$

Proof: The weighted least squares estimates are $\hat{\beta} = \dfrac{SXY}{SXX}$, $\hat{\alpha} = \bar{y}_w - \hat{\beta}\bar{x}_w$, and the corresponding vector of fitted values is

$$\hat{y} = [\bar{y}_w - \frac{SXY}{SXX}\bar{x}_w] + \frac{SXY}{SXX}x \ . \tag{1}$$

Clearly,

$$y - \bar{y}_w = (\hat{y} - \bar{y}_w) + (y - \hat{y}) \ .$$

Since the two parenthesized vectors on the right are orthogonal with respect to $(,)_w$, we have

$$SYY = ||\hat{y} - \bar{y}_w||_w^2 + RSS \quad . \tag{2}$$

Thus, we can write

$$R_w^2 = 1 - \frac{RSS}{SYY} = \frac{||\hat{y} - \bar{y}_w||_w^2}{||y - \bar{y}_w||_w^2} \quad . \tag{3}$$

Using (1) and (2), we can also write

$$RSS = SYY - ||\hat{y} - \bar{y}_w||_w^2 = SYY - \frac{SXY^2}{SXX} \quad . \tag{4}$$

Finally, (4) implies that

$$R_w^2 = 1 - \frac{RSS}{SYY} = \frac{SXY^2}{SXX\ SYY} \quad ,$$

the squared correlation coefficient between x and y with respect to $(\,,)_w$.

Theorem 4: If $\bar{x}$, $\bar{y}$, $||\ ||$, $(\,,)$ are changed to $\bar{x}_w$, $\bar{y}_w$, $||\ ||_w$, $(\,,)_w$, then the Newton's Method algorithm for PRISM described on pages 237-239 of Goldberg (1982) will be a valid algorithm for WPRISM.

Proof: From Theorem 3, for a given τ ,

$$R_w^2(\tau) = \frac{SXY^2(\tau)}{SXX(\tau) SYY} \quad .$$

We seek the value τ such that

$$\frac{d}{d\tau} R_w^2(\tau) = 0 \quad .$$

Since SYY does not depend on TAU , it suffices to find τ such that

$$\frac{d}{d\tau} \frac{SXY^2}{SXX} = 0 \quad .$$

Let $F(\tau) = \dfrac{d}{d\tau} H(\tau)$. Thus, we want

$$0 = \frac{d}{d\tau} \frac{SXY^2}{SXX} = (SXX\ 2SXY\ SFY - SXY^2 2SXF)/SXX^2 |_\tau$$
$$= \frac{2SXY}{SXX^2} (SFY\ SXX - SXY\ SXF)|_\tau \quad ,$$

so it suffices to find τ such that

$$(SFY\ SXX - SXY\ SXF)\ |_\tau = 0 \quad . \tag{1}$$

Now, put $\delta = \tau - \tau_o$, and approximate $x(\tau)$ by

$$x(\tau) = x(\tau_o) + \delta F(\tau_o) \quad .$$

Also, approximate $SFY(\tau)$ by $SFY(\tau) = SFY(\tau_o)$. To this order of approximation, then,

$$SXX(\tau) = SXX(\tau_o) + \delta^2 SFF(\tau_o) + 2\delta\, SXF(\tau_o)$$

$$SXY(\tau) = SXY(\tau_o) + \delta\, SFY(\tau_o)$$

$$SXF(\tau) = SXF(\tau_o) + SFF(\tau_o) \quad .$$

Now, (1) implies that

$$SFY(SXX + \delta^2 SFF + 2\delta\, SXF)\,|_{\tau_o} = (SXY + \delta\, SFY)(SXY + \delta\, SFF)\,|_{\tau_o}$$

and hence

$$[SFY\, SXX + 2\delta\, SFY\, SXF\,]_{\tau_o} = [SXY\, SXF + \delta(SFY\, SXF + SXY\, SFF)]_{\tau_o}$$

and hence, finally,

$$\delta = \left[\frac{SXY\, SXF - SFY\, SXX}{SFY\, SXF - SXY\, SFF}\right]_{\tau_o} \quad .$$

This agrees exactly with Goldberg (1982), p. 239.

Hence if the value of τ after the i'th iteration of Newton's Method is $\tau^{(i)}$, the value after the $i+1$'th iteration is

$$\tau^{(i+1)} = \tau^{(i)} + \delta \quad .$$

AV: The Merits of Using Weighted PRISM When the Period Lengths Differ

The basic PRISM model

$$y = \alpha + \beta H(\tau) + \varepsilon \quad \varepsilon \sim (0, \sigma^2 I_n) \tag{1}$$

is reasonable if the period lengths are nearly the same. If the period lengths are radically different, however, the error terms should not be assumed to have equal variances. Instead, the reasonable assumption is that the variance of ε_i is inversely proportional to the length of period i . This conclusion follows logically if we start with the model (1) with each period having length *one day* , and then average both sides of the equation over blocks of days. Thus, if $\sigma_i^2 =$ [length of period i] $^{-1} = w_i^{-1}$, and W is a diagonal matrix with (i, i) element w_i , then the appropriate model if the period lengths are unequal is

$$y = \alpha + \beta H(\tau) + \varepsilon \quad \varepsilon \sim (0, \sigma^2 W^{-1}) \quad . \tag{2}$$

The most appropriate regression method for fitting such a model is not least squares, but *weighted* least squares with weights proportional to the period lengths. To see this, transform (2) into a model for which ordinary least squares is appropriate: let $\sqrt{W}$ be a diagonal matrix with (i,i) element $\sqrt{w_i}=\sigma_i^{-1}$. Then (2) is equivalent to

$$\sqrt{W}\,y=\sqrt{W}\,\eta(\theta)+\sqrt{W}\,\varepsilon \quad ,\sqrt{W}\,\varepsilon\ \tilde{}\ (0,\sigma^2 I_n) \ ,$$

where

$$\eta(\theta)=\alpha+\beta H(\tau) \ .$$

Since the errors $\sqrt{W}\,\varepsilon$ in this transformed model have equal variances, it is appropriate to fit $\sqrt{W}\,\eta(\theta)$ to the data $\sqrt{W}\,y$ by least squares, thereby minimizing

$$||\sqrt{W}\,(y-\eta(\theta))||^2=\sum_{i=1}^{n} w_i(y_i-\eta_i(\theta))^2 \ .$$

Clearly, this method is equivalent to fitting $\eta(\theta)$ by weighted least squares with weights w_i .

Thus, if one is faced with data having substantially different period lengths, weighted PRISM with weights proportional to the period lengths is preferable to ordinary PRISM. Using PRISM when WPRISM is called for can adversely affect the quality of the estimates, although it will not introduce any systematic bias. Another hazard of using PRISM when WPRISM is called for is that the PRISM standard errors will be incorrect, due to the incorrectness of the PRISM model.

We now derive the theoretical variance-covariance matrix for $\hat{\theta}$ when WPRISM is correctly used. Start with the model

$$y=\eta(\theta)+\varepsilon \quad , \ \varepsilon\ \tilde{}\ (0,\sigma^2 W^{-1}) \ .$$

Carrying out a first order Taylor series expansion of $\eta(\theta)$ about the weighted least squares estimate $\hat{\theta}$, we obtain

$$\eta(\theta)=\eta(\hat{\theta})+\dot{\eta}(\hat{\theta})\,(\theta-\hat{\theta}) \ .$$

Treating $\hat{\theta}$ as fixed, the model becomes

$$y-\eta(\hat{\theta})+\dot{\eta}(\hat{\theta})\,\hat{\theta}=\dot{\eta}(\hat{\theta})\,\theta+\varepsilon \ ,$$

or equivalently,

$$\sqrt{W}\,(y-\eta(\hat{\theta})+\dot{\eta}(\hat{\theta})\,\hat{\theta})=\sqrt{W}\,\dot{\eta}(\hat{\theta})\,\theta+\sqrt{W}\,\varepsilon \ .$$

Since $\sqrt{W}\,\varepsilon\ \tilde{}\ (0,\sigma^2 I_n)$, we have, to this order of approximation,

$$Varcov\ (\hat{\theta}) = \sigma^2(\dot{\eta}(\hat{\theta})^T W\ \dot{\eta}(\hat{\theta}))^{-1}\ . \tag{3}$$

For estimating σ^2 in (3), we should use $\hat{\sigma}_w^2$, the error variance produced by WPRISM.

AVI: Combining RPRISM With the Unequal Period Length Adjustment, OUTWTS

Here, we describe the algorithm used in the latest version of the PRISM software if both the ROBUST and OUTWTS options are set to ON. The method given here simultaneously takes account of unequal period lengths and robustness. This is accomplished by using two sets of weights. The outside weights W_i keep track of the period lengths: W_i is the number of days in period i . The inside weights w_i account for robustness.

The most appropriate regression model can be written as

$$\sqrt{W_i}\,y_i = \sqrt{W_i}\,(\alpha + \beta H_i(\tau)) + \varepsilon_i \quad , \; i = 1, \ldots, n \quad , \tag{1}$$

where $var\ \varepsilon_i = \sigma^2 =$ constant for all i . The algorithm described here can be thought of as giving robust estimates for the model (1):

1) Estimate α , β , τ by WPRISM with weights W_i . Denote the fitted values by $\hat{y}_i$.

2) Use the values $\sqrt{W_i}(y_i - \hat{y}_i)$, which are the "residuals" from the equal-variance model (1), to obtain robustness weights w_i .

3) Obtain new provisional estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ by WPRISM with weights $W_i w_i$. (This is equivalent to fitting the model (1) by weighted least squares with weights w_i .) Denote the fitted values by $\hat{y}_i$.

4) Repeat steps 2) and 3) until the current estimates are sufficiently close to the previous estimates. Use the current estimates as the final estimates.

Note that if all period lengths are equal, then the algorithm described here reduces exactly to the RPRISM (IRLS) algorithm described in Section AII. Note also that the outside weights are never changed.

AVII: Proposed Algorithm To Compensate for Systematic Nonconstant Variability, Unequal Period Lengths, and Isolated Outliers

The algorithm described here has not been implemented in the current version of the PRISM software. It is more general than the algorithm described in Section AVI since it allows for systematic non-constant variability.

1) Run RPRISM with OUTWTS ON. Obtain $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$, $\hat{y}_i$.

2) Given $\hat{y}_i$, compute a new set of outside weights proportional to (length of period i)/ $\hat{y}_i$.

3) Run RPRISM with the current set of outside weights. Obtain new provisional values of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$, $\hat{y}_i$.

4) Repeat steps 2) and 3) until convergence is reached

5) Stop. Use the current values of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ as the final estimates.

Appendix B. Summary of New Additions to the PRISM Software

The developments described in this paper have been incorporated in an earlier research version of the PRISM software. Specifically, this preliminary version of Robust PRISM now has two new options, controlled by the 'SET' command. The ROBUST option, which is OFF by default, causes the execution of the Robust PRISM (RPRISM) algorithm. The OUTWTS option, which is also OFF by default, runs weighted PRISM (WPRISM), with weights proportional to the period lengths. If desired, both options may be set to ON. This would simultaneously provide the user with robustness and with the unequal period-length adjustment. If both options are set to OFF (the default settings), then the ordinary PRISM algorithm will be executed.

All runs of RPRISM done for this paper used six iterations of WPRISM (see schematic for RPRISM, Figure 1). Subsequently, the algorithm has been revised to stop if the weights converge (to within a tolerance of .01) before six iterations. If the ROBUST option is used, the initial and final (inside) weights for each house will be written to Fortran Unit 11.

Appendix C: Using Weighted PRISM to Compensate for Another Kind of Nonconstant Variability

In developing models for electrical consumption, Latta (1983) has suggested that the variance of the consumption may be approximately proportional to the fitted consumption value. Thus, according to Latta's proposal the variance is nonconstant, and increases with fitted consumption. Although Latta's models differ from the PRISM model in many ways, the assumed pattern of variability may be a good approximation to the truth for PRISM as well. (Indeed, this type of pattern has often been proposed by statisticians as a realistic one for many commonly encountered regression data sets.)

To formalize the idea, write the (modified) PRISM model as

$$y_i = \alpha_i + \beta H_i(\tau) + \epsilon_i$$

where the ϵ_1 are independent zero mean random variables with variances σ_i^2, to be determined. For simplicity, suppose that the period lengths are all identical. Now, assume that $\text{var}(y_i) = \sigma_i^2$ is proportional to $\alpha + \beta H_i(\tau)$, the (theoretical) average consumption according to the true model. This is a reasonable assumption, under which variance increases with average consumption. Thus, if α, β, and τ were known, the optimal fitting method would be weighted least squares (i.e., WPRISM), with weights inversely proportional to $\alpha + \beta H_i(\tau)$. Unfortunately, α, β, and τ are unknown, and so the best we can do is to obtain preliminary estimates $\hat{\alpha}$, $\hat{\beta}$, $\hat{\tau}$ (e.g., by using ordinary PRISM), and then to assume that $\text{var}(y_i)$ is proportional to the <u>fitted</u> consumption, $\hat{y}_i = \hat{\alpha} + \hat{\beta} H_i(\hat{\tau})$. Thus, the second stage in the fitting process would be to apply WPRISM to the original data, with weights inversely proportional to $\hat{y}_i$. This would yield new estimates, as well as new fitted values.

We could stop here, and use the current values as the final estimates. Alternatively, if desired, we could iterate the fitting process, at each stage computing new weights from the present fit and then running WPRISM with these weights. The iterations could be stopped when the fitted values at a given stage are sufficiently close to the fitted values at the previous stage.

Note that the iterative scheme proposed above is analogous, though not identical, to the iterations used in calculating RPRISM estimates. The difference is that the weights computed for RPRISM are designed to directly compensate for the effects of outliers, while the weights computed for the present scheme are designed to compensate for an assumed global pattern of variability.

It would be relatively easy to incorporate the compensation discussed in this section with the previously described compensation for unequal period lengths (OUTWTS) and the compensation for outliers (RPRISM). A specification of the complete algorithm is given in Appendix AVI.

Tables and Figures

Note: To facilitate continued review of this methodology development, extremely detailed results are included in some of the tables and figures (in the form of Univariate summaries from SAS, house-by-house PRISM output, and individual-house plots of consumption data). Those tables and figures containing concise summaries of the detailed results are indicated by asterisks in the text. For a cursory reading of the text, the reader may wish to emphasize the (asterisked) summaries.

Table 1. Summary of median accuracy measures from ordinary PRISM[a].

Heating fuel:	Electricity (i)	Gas (ii)	Oil (iii)
# houses	50	276	207
R^2	0.972	0.990	0.971
$se(\alpha)/\alpha$	0.123	0.160	0.476[b]
$se(\beta)/\beta$	0.098	0.076	0.105
$se(\tau)$ (°C)	1.7	1.2	3.0
se(NAC)/NAC	0.030	0.025	0.073

a. Median R^2 values and standard errors from ordinary PRISM applied to three New Jersey samples of houses are shown:

i) a set of 50 houses from our General Public Utilities data base, which are heated but not cooled by electricity (Stram and Fels, 1986)

ii) gas-heated houses in the Modular Retrofit Experiment (a pre- and a post-retrofit period for each of 138 houses) (Dutt et al., 1986)

iii) a set of 207 oil-heated houses (Fels et al., 1986).

Standard errors of α, β, and NAC are expressed as fractions of the estimate:

b. Since all houses in the other two samples have their water heated by the space heating fuel, the median of $se(\alpha)/\alpha$ for the oil sample corresponds to the subset of 133 houses with oil-fired water heaters.

Results in this table are reproduced from an earlier paper (Fels et al., 1986, Table 2).

Table 2. By-house PRISM results for 50 electrically heated homes in New Jersey (Elec-HO sample). Units for PRISM parameters are as follows: τ(TREF) = °F; α(BASELEVEL) = kWh/day; β(SLOPE) = kWh/°F-day; NAC(Normalized Annual Consumption) = kWh/year; βH_o(HEATING PART) = kWh/year.

UNIT ID	TIME PERIOD	# PDS	# DAYS	RAW CONS	# ITS	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR	HEATING PART
J3	12/ 4/78-12/ 4/79	12	365	15889	3	0.972	59.7(2.8)	18.61(2.58)	2.528(0.272)	16279.92(591.68)	9483.9(782.2)
J4	12/20/78-12/20/79	12	365	23279	5	0.993	50.9(1.2)	29.06(1.98)	5.975(0.339)	23579.34(503.88)	12964.8(565.0)
J5	12/26/78-12/26/79	12	365	41639	4	0.991	70.0(2.3)	51.42(5.37)	3.788(0.179)	42208.88(646.24)	23428.2(1726.6)
J6	12/11/78-12/11/79	12	365	27979	3	0.993	64.5(1.6)	26.33(2.77)	3.932(0.195)	28526.00(513.06)	18908.7(878.2)
J7	12/19/78-12/19/79	12	365	28519	3	0.966	61.9(3.2)	51.19(3.20)	2.434(0.269)	28935.67(668.48)	10238.3(981.3)
J10	12/22/78-12/24/79	12	367	30339	3	0.976	61.9(2.7)	44.26(3.73)	3.426(0.321)	30581.97(779.55)	14417.0(1137.5)
J11	12/27/78-12/28/79	12	366	17989	3	0.936	61.6(4.5)	28.33(3.55)	1.929(0.303)	18358.97(739.62)	8010.8(1088.9)
J16	12/14/78-12/14/79	12	365	17949	3	0.989	60.4(1.7)	23.12(1.73)	2.553(0.164)	18379.12(374.47)	9934.9(528.6)
J17	12/27/78-12/28/79	12	366	22051	3	0.983	66.0(2.7)	27.90(3.24)	2.380(0.171)	22474.81(512.06)	12285.4(1043.4)
J18	12/22/78-12/24/79	12	367	13279	4	0.904	61.0(5.3)	17.79(3.65)	1.732(0.339)	13455.32(798.25)	6957.7(1100.1)
J19	12/19/78-12/19/79	12	365	59429	7	0.970	82.0(13.3)	19.79(63.72)	5.313(0.371)	60022.26(1860.89)	52793.1(23385.2)
J20	12/ 5/78-12/ 5/79	12	365	8575	5	0.921	57.5(4.7)	7.22(2.88)	1.840(0.362)	8739.71(686.59)	6102.8(864.1)
J21	12/ 1/78-12/ 3/79	12	367	38299	4	0.973	58.0(2.4)	55.56(4.95)	5.377(0.529)	38613.22(1157.20)	18320.3(1478.7)
J22	12/13/78-12/13/79	12	365	43169	4	0.971	66.1(3.5)	28.66(11.35)	6.530(0.615)	44362.91(1819.59)	33894.6(3636.4)
J23	12/12/78-12/12/79	12	365	31219	4	0.933	59.6(4.6)	38.92(7.57)	4.725(0.840)	31839.25(1705.52)	17622.4(2324.8)
J24	11/30/78-11/30/79	12	365	21999	3	0.966	64.4(3.4)	24.19(4.52)	2.866(0.297)	22520.50(853.35)	13686.5(1422.7)
J25	12/ 5/78-12/ 5/79	12	365	30739	3	0.958	64.7(4.0)	36.60(6.41)	3.693(0.457)	31267.29(1222.00)	17900.4(2024.4)
J26	12/ 7/78-12/ 7/79	12	365	26059	3	0.976	60.2(2.5)	26.62(4.27)	4.367(0.411)	26556.09(944.24)	16833.8(1284.8)
J27	12/11/78-12/11/79	12	365	20219	3	0.983	59.0(2.2)	24.76(2.47)	3.199(0.274)	20575.04(557.01)	11533.3(757.4)
J28	12/28/78-12/28/79	12	365	10476	11	0.062	23.0(43.2)	28.06(2.47)	1.486(7.264)	10355.92(854.05)	108.0(562.5)
J29	12/20/78-12/20/79	12	365	19059	3	0.985	60.9(2.0)	25.57(2.02)	2.529(0.184)	19458.17(446.53)	10117.6(611.0)
J30	12/ 1/78-12/ 3/79	12	367	33799	3	0.988	65.2(2.1)	35.76(4.32)	4.259(0.264)	34259.31(758.81)	21196.7(1371.4)
J32	12/ 7/78-12/ 7/79	12	365	28699	4	0.988	64.0(2.0)	30.41(3.31)	3.861(0.250)	29201.95(648.64)	18093.6(1025.6)
J33	12/26/78-12/26/79	12	365	25229	4	0.971	67.4(3.9)	24.25(6.30)	3.080(0.293)	25849.05(900.95)	16992.3(2014.5)
J34	12/28/78-12/28/79	12	365	22009	3	0.918	63.3(5.3)	27.33(6.57)	2.772(0.467)	22549.16(1278.83)	12565.6(2045.5)
J38	12/ 6/78-12/ 6/79	12	365	25479	3	0.972	61.2(2.8)	31.33(3.98)	3.564(0.363)	25926.55(860.10)	14484.4(1212.9)
J39	12/13/78-12/13/79	12	365	27269	4	0.994	65.8(1.5)	14.94(3.14)	4.420(0.192)	28067.22(543.46)	22609.7(998.0)
J40	12/12/78-12/12/79	12	365	26599	2	0.991	63.1(1.7)	33.99(2.47)	3.284(0.187)	27137.64(474.96)	14724.3(770.9)
J41	12/11/78-12/11/79	12	365	24779	3	0.868	55.4(6.5)	41.94(6.42)	3.353(0.964)	25087.87(1514.06)	9770.7(1912.1)
J43	12/15/78-12/17/79	12	367	22909	8	0.915	45.2(4.6)	35.94(5.78)	7.081(2.070)	22906.89(1568.98)	9780.4(1614.0)
J44	12/19/78-12/19/79	12	365	20339	3	0.990	59.4(1.6)	29.04(1.65)	2.735(0.166)	20704.90(375.28)	10097.4(501.7)
J45	12/ 4/78-12/ 4/79	12	365	26869	3	0.969	64.7(3.4)	28.29(5.27)	3.532(0.369)	27479.96(993.53)	17147.0(1663.6)
J46	12/27/78-12/28/79	12	366	20109	3	0.976	68.1(3.4)	20.65(4.40)	2.282(0.187)	20508.37(626.22)	12965.6(1409.7)
J47	12/19/78-12/19/79	12	365	24099	5	0.873	70.6(9.5)	24.98(13.95)	2.435(0.467)	24555.00(1692.84)	15429.5(4487.7)
J48	12/ 5/78-12/ 5/79	12	365	35839	3	0.975	61.4(2.6)	47.79(4.95)	4.638(0.448)	36488.96(1070.63)	19034.4(1505.5)
J49	12/15/78-12/17/79	12	367	22599	4	0.948	59.0(3.8)	43.19(2.70)	1.942(0.285)	22777.74(604.94)	7000.9(827.6)
J50	11/30/78-11/30/79	12	365	22259	3	0.996	69.1(1.3)	16.66(2.21)	2.793(0.083)	22720.55(305.18)	16636.0(700.6)
J51	12/13/78-12/13/79	12	365	22399	4	0.956	51.6(3.7)	45.07(2.32)	2.695(0.487)	22588.97(576.72)	6125.4(706.0)
J54	12/ 4/78-12/ 4/79	12	365	18980	2	0.965	62.8(3.3)	27.25(2.97)	2.134(0.242)	19349.84(606.09)	9397.0(918.0)
J55	12/18/78-12/18/79	12	365	36379	3	0.986	63.5(2.1)	41.21(4.62)	4.852(0.323)	37197.82(883.89)	22144.9(1439.4)
J57	12/26/78-12/26/79	12	365	28619	5	0.982	66.4(3.0)	34.22(4.67)	3.184(0.245)	29258.22(720.28)	16758.4(1503.1)
J58	12/28/78-12/28/79	12	365	18509	4	0.997	55.9(0.9)	21.18(1.09)	3.741(0.136)	18994.35(263.81)	11256.9(319.7)
J60	12/15/78-12/17/79	12	367	23978	3	0.973	60.9(2.8)	32.74(3.37)	3.078(0.314)	24282.05(724.00)	12323.7(1024.1)
J62	12/26/78-12/26/79	12	365	29069	4	0.956	67.0(4.8)	37.84(7.37)	2.921(0.342)	29619.67(1053.95)	15798.1(2356.6)
J63	12/18/78-12/18/79	12	365	25399	3	0.856	58.8(6.4)	33.26(9.32)	3.918(0.981)	26108.77(2147.61)	13959.0(2782.6)
J64	12/26/78-12/26/79	12	365	28849	4	0.943	70.5(6.2)	23.73(12.26)	3.295(0.409)	29461.89(1476.04)	20793.2(3943.6)
J65	12/ 4/78-12/ 4/79	12	365	21759	3	0.993	61.6(1.4)	31.50(1.43)	2.569(0.127)	22162.41(305.51)	10656.5(434.6)
J67	12/22/78-12/24/79	12	367	42739	5	0.966	68.0(4.2)	72.01(6.77)	2.931(0.291)	42895.33(934.81)	16592.0(2162.4)
J69	12/ 6/78-12/ 6/79	12	365	27319	2	0.969	62.2(3.0)	32.92(4.67)	3.703(0.392)	27850.84(979.37)	15825.4(1440.7)
J70	12/18/78-12/18/79	12	365	24389	3	0.997	65.0(1.1)	23.36(1.67)	3.347(0.109)	24979.45(308.99)	16447.9(525.6)

Table 3. By-house RPRISM results for Elec-HO sample. See Table 2 caption.

UNIT ID	TIME PERIOD	# PDS	# DAYS	RAW CONS	# ITS	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR	HEATING PART
J3	12/ 4/78-12/ 4/79	12	365	15889	2	0.977	59.1(1.8)	18.73(1.81)	2.629(0.191)	16397.70(414.87)	9558.0(548.5)
J4	12/20/78-12/20/79	12	365	23279	1	0.995	51.1(0.9)	29.44(1.59)	5.808(0.258)	23474.59(405.86)	12723.2(465.6)
J5	12/26/78-12/26/79	12	365	41639	2	0.992	69.7(2.5)	51.73(5.57)	3.810(0.208)	42138.15(733.23)	23243.8(1772.7)
J6	12/11/78-12/11/79	12	365	27979	2	0.993	64.8(1.6)	26.07(2.76)	3.871(0.195)	28371.84(512.17)	18850.6(876.7)
J7	12/19/78-12/19/79	12	365	28519	1	0.966	61.9(3.2)	51.19(3.20)	2.434(0.269)	28935.67(668.49)	10238.3(981.3)
J10	12/22/78-12/24/79	12	367	30339	3	0.976	62.0(2.9)	44.24(3.95)	3.403(0.340)	30537.96(825.12)	14380.1(1204.0)
J11	12/27/78-12/28/79	12	366	17989	2	0.959	63.8(3.9)	27.91(3.05)	1.810(0.220)	18587.02(586.40)	8394.0(953.9)
J16	12/14/78-12/14/79	12	365	17949	1	0.990	60.7(1.9)	23.02(1.89)	2.524(0.178)	18417.42(407.55)	10007.6(575.3)
J17	12/27/78-12/28/79	12	366	22051	1	0.995	65.5(1.3)	28.19(1.42)	2.378(0.084)	22284.07(242.52)	11988.1(454.3)
J18	12/22/78-12/24/79	12	367	13279	3	0.926	58.0(3.8)	18.51(2.79)	1.877(0.300)	13154.87(655.45)	6395.9(820.3)
J19	12/19/78-12/19/79	12	365	59429	2	0.992	75.4(1.8)	53.68(6.99)	5.354(0.146)	61013.75(611.16)	41406.6(2424.8)
J20	12/ 5/78-12/ 5/79	12	365	8575	2	0.931	58.8(4.8)	7.37(2.97)	1.739(0.340)	8906.63(702.52)	6213.9(886.3)
J21	12/ 1/78-12/ 3/79	12	367	38299	2	0.984	58.0(1.9)	54.82(3.84)	5.292(0.411)	38055.41(898.63)	18031.9(1148.3)
J22	12/13/78-12/13/79	12	365	43169	2	0.974	65.5(2.9)	28.88(9.52)	6.827(0.581)	44932.43(1645.80)	34383.5(3022.4)
J23	12/12/78-12/12/79	12	365	31219	1	0.965	64.5(3.8)	33.21(6.97)	4.200(0.487)	32326.46(1287.41)	20198.3(2210.4)
J24	11/30/78-11/30/79	12	365	21999	2	0.967	64.2(3.4)	24.29(4.55)	2.880(0.299)	22482.84(858.52)	13612.2(1431.4)
J25	12/ 5/78-12/ 5/79	12	365	30739	1	0.958	64.7(4.3)	36.55(6.95)	3.698(0.494)	31253.70(1323.71)	17904.9(2192.9)
J26	12/ 7/78-12/ 7/79	12	365	26059	2	0.982	58.7(2.3)	27.57(3.73)	4.708(0.431)	26805.00(862.50)	16735.9(1118.8)
J27	12/11/78-12/11/79	12	365	20219	2	0.992	58.0(1.2)	24.73(1.32)	3.262(0.157)	20160.40(301.23)	11126.5(398.1)
J28	12/28/78-12/28/79	12	365	10476	3	0.179	27.0(15.9)	27.02(1.48)	1.172(2.020)	10052.43(495.75)	184.1(292.4)
J29	12/20/78-12/20/79	12	365	19059	1	0.987	60.7(2.0)	25.65(2.06)	2.536(0.188)	19406.60(455.07)	10038.0(622.7)
J30	12/ 1/78-12/ 3/79	12	367	33799	2	0.989	65.0(2.2)	36.61(4.59)	4.258(0.280)	34362.32(806.05)	20990.7(1456.8)
J32	12/ 7/78-12/ 7/79	12	365	28699	2	0.992	63.3(1.6)	30.58(2.75)	3.897(0.208)	28813.80(539.42)	17644.4(852.9)
J33	12/26/78-12/26/79	12	365	25229	2	0.990	66.6(2.2)	24.77(3.37)	3.108(0.177)	25568.60(519.50)	16520.2(1084.1)
J34	12/28/78-12/28/79	12	365	22009	2	0.944	61.0(3.2)	28.24(3.87)	2.881(0.326)	21911.83(815.99)	11597.9(1185.1)
J38	12/ 6/78-12/ 6/79	12	365	25479	2	0.975	61.2(3.0)	31.45(4.36)	3.592(0.397)	26052.31(941.58)	14566.3(1327.8)
J39	12/13/78-12/13/79	12	365	27269	2	0.996	66.3(1.2)	13.88(2.64)	4.369(0.143)	27994.51(423.25)	22923.1(845.9)
J40	12/12/78-12/12/79	12	365	26599	1	0.991	63.2(1.7)	33.97(2.36)	3.248(0.178)	27044.83(453.61)	14638.0(736.2)
J41	12/11/78-12/11/79	12	365	24779	2	0.877	55.7(5.9)	41.87(5.64)	3.292(0.847)	25058.34(1330.06)	9765.6(1679.7)
J43	12/15/78-12/17/79	12	367	22909	2	0.984	53.5(1.0)	31.75(1.51)	5.116(0.230)	24837.74(367.87)	13242.7(430.8)
J44	12/19/78-12/19/79	12	365	20339	1	0.994	58.1(1.0)	29.33(1.14)	2.851(0.121)	20508.72(265.59)	9796.5(339.1)
J45	12/ 4/78-12/ 4/79	12	365	26869	2	0.980	67.6(2.7)	25.45(4.59)	3.187(0.223)	27033.50(701.28)	17738.8(1458.6)
J46	12/27/78-12/28/79	12	366	20109	2	0.979	69.2(3.3)	20.00(4.34)	2.207(0.163)	20457.43(584.90)	13153.4(1380.6)
J47	12/19/78-12/19/79	12	365	24099	2	0.959	68.7(3.4)	26.68(4.34)	2.306(0.184)	23197.67(616.69)	13451.5(1387.4)
J48	12/ 5/78-12/ 5/79	12	365	35839	2	0.983	62.6(2.4)	47.68(4.45)	4.445(0.371)	36846.83(924.99)	19430.2(1376.1)
J49	12/15/78-12/17/79	12	367	22599	2	0.957	58.2(3.5)	43.26(2.59)	2.043(0.290)	22841.31(592.54)	7041.5(777.2)
J50	11/30/78-11/30/79	12	365	22259	1	0.996	69.1(1.3)	16.66(2.21)	2.793(0.083)	22720.55(305.09)	16636.0(700.3)
J51	12/13/78-12/13/79	12	365	22399	2	0.966	51.8(3.6)	45.20(2.21)	2.666(0.464)	22674.26(548.98)	6164.9(672.1)
J54	12/ 4/78-12/ 4/79	12	365	18980	2	0.971	62.3(2.3)	27.37(2.07)	2.165(0.169)	19322.86(422.26)	9326.7(639.6)
J55	12/18/78-12/18/79	12	365	36379	2	0.986	63.6(2.4)	41.16(5.25)	4.818(0.366)	37145.00(1003.94)	22111.4(1634.9)
J57	12/26/78-12/26/79	12	365	28619	1	0.982	66.4(3.0)	34.22(4.67)	3.184(0.245)	29258.22(720.21)	16758.4(1503.0)
J58	12/28/78-12/28/79	12	365	18509	3	0.997	56.0(0.8)	21.19(1.14)	3.721(0.133)	19010.09(273.78)	11269.7(330.9)
J60	12/15/78-12/17/79	12	367	23978	2	0.988	61.1(1.8)	32.85(2.16)	2.947(0.187)	23909.30(444.77)	11911.5(666.0)
J62	12/26/78-12/26/79	12	365	29069	2	0.969	67.2(4.9)	36.47(7.43)	2.894(0.345)	29146.49(1062.41)	15826.5(2375.5)
J63	12/18/78-12/18/79	12	365	25399	2	0.911	60.2(4.8)	33.48(7.46)	3.755(0.666)	26730.18(1607.29)	14500.0(2263.6)
J64	12/26/78-12/26/79	12	365	28849	2	0.961	68.7(4.9)	24.99(9.23)	3.452(0.387)	29277.95(1271.28)	20150.2(2957.4)
J65	12/ 4/78-12/ 4/79	12	365	21759	2	0.994	61.8(1.4)	31.32(1.50)	2.575(0.133)	22232.21(319.81)	10794.1(454.9)
J67	12/22/78-12/24/79	12	367	42739	1	0.971	67.8(4.0)	72.09(5.96)	2.909(0.283)	42686.69(861.08)	16357.0(1902.7)
J69	12/ 6/78-12/ 6/79	12	365	27319	2	0.969	62.1(3.2)	32.94(5.04)	3.716(0.423)	27867.24(1056.82)	15836.1(1554.6)
J70	12/18/78-12/18/79	12	365	24389	2	0.997	65.0(1.2)	23.37(1.79)	3.348(0.117)	24974.71(332.61)	16438.4(565.8)

Table 4. Summary statistics of PRISM vs. RPRISM parameters for Elec-HO sample.

	MEDIANS		MEANS		
	REGULAR	ROBUST	REGULAR	ROBUST	
ALPHA	28.85	29.39	31.53	32.03	kWh/day
BETA	3.24	3.22	3.42	3.36	kWh/°F-day
BHO	14451	14440	15043	14844	kWh/year
NAC	25469	25314	26561	26544	kWh/year
R-SQUARE	.972	.981	.945	.959	
CV ALPHA	.123	.108	.217	.127	
CV BETA	.098	.077	.204	.120	
CV BHO	.090	.075	.209	.111	
CV NAC	.030	.024	.034	.027	

Table 5. By-house percent differences for each estimate for Elec-HO sample. Percent difference is defined as:
[(RPRISM estimate-PRISM estimate)/PRISM estimate] * 100.

HOUSELET	PCALPHA	PCBETA	PCNAC	PCBHO
J3	0.645	3.995	0.7235	0.781
J4	1.308	-2.795	-0.4442	-1.864
J5	0.603	0.581	-0.1676	-0.787
J6	-0.987	-1.551	-0.5404	-0.307
J7	0.000	0.000	0.0000	0.000
J10	-0.045	-0.671	-0.1439	-0.256
J11	-1.483	-6.169	1.2422	4.784
J16	-0.433	-1.136	0.2084	0.732
J17	1.039	-0.084	-0.8487	-2.420
J18	4.047	8.372	-2.2329	-8.075
J19	171.248	0.772	1.6519	-21.568
J20	2.078	-5.489	1.9099	1.820
J21	-1.332	-1.581	-1.4446	-1.574
J22	0.768	4.548	1.2838	1.442
J23	-14.671	-11.111	1.5302	14.617
J24	0.413	0.488	-0.1672	-0.543
J25	-0.137	0.135	-0.0435	0.025
J26	3.569	7.809	0.9373	-0.582
J27	-0.121	1.969	-2.0153	-3.527
J28	-3.706	-21.131	-2.9306	70.463
J29	0.313	0.277	-0.2650	-0.787
J30	2.377	-0.023	0.3007	-0.972
J32	0.559	0.932	-1.3292	-2.483
J33	2.144	0.909	-1.0850	-2.778
J34	3.330	3.932	-2.8264	-7.701
J38	0.383	0.786	0.4851	0.565
J39	-7.095	-1.154	-0.2591	1.386
J40	-0.059	-1.096	-0.3420	-0.586
J41	-0.167	-1.819	-0.1177	-0.052
J43	-11.658	-27.750	8.4291	35.400
J44	0.999	4.241	-0.9475	-2.980
J45	-10.039	-9.768	-1.6247	3.451
J46	-3.148	-3.287	-0.2484	1.448
J47	6.805	-5.298	-5.5277	-12.820
J48	-0.230	-4.161	0.9808	2.079
J49	0.162	5.201	0.2791	0.580
J50	0.000	0.000	0.0000	0.000
J51	0.288	-1.076	0.3776	0.645
J54	0.440	1.453	-0.1394	-0.748
J55	-0.121	-0.701	-0.1420	-0.151
J57	0.000	0.000	0.0000	0.000
J58	0.047	-0.535	0.0829	0.114
J60	0.336	-4.256	-1.5351	-3.345
J62	-3.621	-0.924	-1.5975	0.180
J63	0.661	-4.160	2.3801	3.876
J64	5.310	4.765	-0.6243	-3.092
J65	-0.571	0.234	0.3149	1.291
J67	0.111	-0.751	-0.4864	-1.416
J69	0.061	0.351	0.0589	0.068
J70	0.043	0.030	-0.0190	-0.058

Table 6. Summary statistics of percent differences of PRISM vs. RPRISM parameters for Elec-HO sample. (See Figures 2-5.)

	MIN	Q1	MED	Q3	MAX	MEAN
ALPHA	-14.7	-.28	.086	.825	171	3.01
BETA	-27.8	-2.1	-.054	.816	8.37	-1.33
NAC	-5.5	-.87	-.141	.331	8.43	-.138
HEATING	-22	-1.6	-.055	.91	70.5	1.29

Table 7. By-house summary of the ratio of the coefficients of variation (CVR) for PRISM vs. RPRISM estimates for Elec-HO sample. CVR = CV(PRISM estimate)/CV(RPRISM estimate).

HOUSELET	CVRALPHA	CVRBETA	CVRNAC	CVRBHO
J3	1.4346	1.48098	1.43650	1.43721
J4	1.2616	1.27723	1.23600	1.19087
J5	0.9699	0.86557	0.87988	0.96633
J6	0.9937	0.98449	0.99632	0.99863
J7	1.0000	1.00000	0.99999	1.00000
J10	0.9439	0.93778	0.94341	0.94235
J11	1.1467	1.29231	1.27696	1.19613
J16	0.9114	0.91088	0.92075	0.92555
J17	2.3054	2.03400	2.09349	2.24114
J18	1.3612	1.22460	1.19067	1.23281
J19	24.7267	2.56071	3.09515	7.56411
J20	0.9898	1.00626	0.99599	0.99270
J21	1.2719	1.26676	1.26914	1.26746
J22	1.2014	1.10666	1.11979	1.22050
J23	0.9267	1.53320	1.34504	1.20549
J24	0.9975	0.99816	0.99232	0.98853
J25	0.9210	0.92635	0.92276	0.92339
J26	1.1856	1.02806	1.10503	1.14169
J27	1.8689	1.77959	1.81185	1.83543
J28	1.6071	2.83618	1.67226	3.27926
J29	0.9837	0.98143	0.97863	0.97349
J30	0.9635	0.94264	0.94422	0.93223
J32	1.2104	1.21313	1.18649	1.17263
J33	1.9095	1.67042	1.71545	1.80660
J34	1.7542	1.48884	1.52292	1.59309
J38	0.9163	0.92154	0.91790	0.91863
J39	1.1050	1.32717	1.28069	1.19616
J40	1.0460	1.03905	1.04349	1.04100
J41	1.1364	1.11743	1.13700	1.13776
J43	3.3816	6.50247	4.62454	5.07280
J44	1.4618	1.43009	1.39962	1.43542
J45	1.0329	1.49308	1.39372	1.17991
J46	0.9819	1.10953	1.06799	1.03587
J47	3.4330	2.40358	2.59330	2.81995
J48	1.1098	1.15730	1.16880	1.11678
J49	1.0442	1.03387	1.02378	1.07102
J50	1.0000	1.00000	1.00029	1.00043
J51	1.0528	1.03827	1.05450	1.05721
J54	1.4411	1.45275	1.43335	1.42453
J55	0.8789	0.87633	0.87917	0.87909
J57	1.0000	1.00000	1.00010	1.00007
J58	0.9566	1.01709	0.96438	0.96725
J60	1.5654	1.60768	1.60282	1.48626
J62	0.9560	0.98214	0.97619	0.99383
J63	1.2576	1.41169	1.36797	1.27692
J64	1.3988	1.10720	1.15382	1.29223
J65	0.9479	0.95712	0.95829	0.96771
J67	1.1372	1.02055	1.08034	1.12039
J69	0.9272	0.92997	0.92726	0.92736
J70	0.9334	0.93190	0.92881	0.92841

Table 8. Summary statistics of CVRs of PRISM vs. RPRISM parameters for Elec-HO sample. (See Figures 7-10.)

	MIN	Q1	MED	Q3	MAX	MEAN
ALPHA	.88	.97	1.08	1.41	24.7	1.74
BETA	.87	.98	1.11	1.46	6.50	1.36
NAC	.88	.98	1.11	1.40	4.62	1.31
HEATING	.88	.98	1.13	1.33	7.56	1.45

Table 9. PRISM and RPRISM results for House J19 from Elec-HO sample.

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                          PRISM

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ESTIMATION FOR HOUSE J19 , PERIOD: DEC 19, 1978 TO DEC 19, 1979

           REFERENCE        HEATING    BASE       NORM ANNUAL   R-SQUARE   NUM
           TEMPERATURE      SLOPE      LEVEL      CONSUMPTION              OF OBS

 ESTIMATES:    82.00         5.3130    19.7923     60022.2617    0.9699     12
(STD ERRS)    (13.34)       (0.3713) (63.7163)    (1860.8901)

HEATING PART OF NAC:     52793.1328 (23385.1953) % OF NAC  88.0  ENERGY UNITS:

                                INTERCEPT     SLOPE   REFERENCE TEMP
     CORRELATION      INTERCEPT  1.0
     MATRIX FOR       SLOPE      0.4270        1.0
     ESTIMATES        REF TEMP  -0.9882       -0.5433       1.0

     ERROR VARIANCE:     308.3   NUMBER OF ITERATIONS:  7

TECHNICAL CODES:  G  RMF 0.000 RTF 0.000 FBT 0.000 MKP 0.000 TRT 0.000
*******************************************************************************
                          RPRISM

ESTIMATION FOR HOUSE J19 , PERIOD: DEC 19, 1978 TO DEC 19, 1979

           REFERENCE        HEATING    BASE       NORM ANNUAL   R-SQUARE   NUM
           TEMPERATURE      SLOPE      LEVEL      CONSUMPTION              OF OBS

 ESTIMATES:    75.38         5.3545   53.6814      61013.7539    0.9920     12
(STD ERRS)    (1.83)        (0.1456)  (6.9918)     (611.1587)

HEATING PART OF NAC;     41406.6406 (2424.7996) % OF NAC  67.9  ENERGY UNITS:

                                INTERCEPT     SLOPE   REFERENCE TEMP
     CORRELATION      INTERCEPT  1.0
     MATRIX FOR       SLOPE      0.3828        1.0
     ESTIMATES        REF TEMP  -0.9172       -0.6746       1.0

     ERROR VARIANCE:     55.45   NUMBER OF ITERATIONS:  2

TECHNICAL CODES:     RMF 0.000 RTF 0.000 FBT 0.000 MKP 0.000 TRT 0.000
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Table 10. PRISM and RPRISM results for House J44 from Elec-HO sample. House identifiers in column 1 are defined as follows: J44 - original data were run; K44 - Point D from data was inflated from 1940 kWh to 3000 kWh; 044 - Point D was removed from the data.

	METHOD	PRE OR POST	SAMP TYPE	TIME PERIOD	# PDS	# DAYS	RAW CONS	# ITS	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR
J44	PRISM	CELL		12/19/78-12/19/79	12	365	20339	3	0.990	59.4(1.6)	29.04(1.65)	2.735(0.166)	20704.90(375.28)
J44	RPRISM	CELL		12/19/78-12/19/79	12	365	20339	2	0.994	58.1(1.0)	29.33(1.14)	2.851(0.121)	20508.59(265.58)
K44	PRISM	CELL		12/19/78-12/19/79	12	365	21399	3	0.882	66.2(7.3)	26.58(8.61)	2.318(0.454)	21776.91(1393.56)
K44	RPRISM	CELL		12/19/78-12/19/79	12	365	21399	1	0.983	58.1(1.0)	29.33(1.14)	2.850(0.121)	20508.74(265.59)
044	PRISM	CELL		12/19/78-12/19/79	11	335	18399	5	0.998	56.9(0.9)	29.59(0.81)	2.981(0.106)	20316.54(206.61)
044	RPRISM	CELL		12/19/78-12/19/79	11	335	18399	2	0.998	56.8(1.2)	29.62(1.10)	2.974(0.143)	20287.43(279.46)

Table 11. By-house initial and final weights from RPRISM analysis of Elec-HO sample.

HOUSE J3												
INITIAL	1.00	0.38	0.41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.73
FINAL	1.00	0.41	0.33	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.72
HOUSE J4												
INITIAL	0.52	0.85	1.00	0.75	1.00	1.00	1.00	0.94	1.00	1.00	1.00	1.00
FINAL	0.36	1.00	1.00	0.61	1.00	1.00	1.00	0.75	1.00	1.00	1.00	1.00
HOUSE J5												
INITIAL	1.00	1.00	1.00	0.84	1.00	1.00	1.00	1.00	1.00	0.96	0.76	1.00
FINAL	1.00	1.00	1.00	0.84	1.00	1.00	1.00	1.00	1.00	0.85	0.68	1.00
HOUSE J6												
INITIAL	1.00	0.66	1.00	1.00	1.00	0.79	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	0.59	1.00	1.00	1.00	0.84	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J7												
INITIAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J10												
INITIAL	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J11												
INITIAL	1.00	1.00	0.67	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.48	1.00
FINAL	1.00	1.00	0.74	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.40	1.00
HOUSE J16												
INITIAL	0.94	1.00	1.00	0.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	0.93	1.00	1.00	0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J17												
INITIAL	0.74	1.00	0.23	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.40	1.00
FINAL	0.84	1.00	0.18	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.38	1.00
HOUSE J18												
INITIAL	1.00	1.00	0.52	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.72	0.99
FINAL	1.00	1.00	0.49	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J19												
INITIAL	1.00	1.00	0.52	1.00	0.32	1.00	1.00	0.29	1.00	1.00	1.00	0.24
FINAL	1.00	1.00	0.22	1.00	0.13	1.00	1.00	0.15	0.54	1.00	1.00	0.14
HOUSE J20												
INITIAL	1.00	1.00	1.00	1.00	0.72	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	0.95	1.00	1.00	1.00	0.57	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J21												
INITIAL	0.44	1.00	1.00	1.00	0.71	1.00	1.00	0.70	1.00	1.00	1.00	1.00
FINAL	0.37	1.00	1.00	1.00	0.72	1.00	1.00	0.61	1.00	1.00	1.00	1.00
HOUSE J22												
INITIAL	1.00	0.64	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	0.55	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J23												
INITIAL	1.00	1.00	1.00	1.00	1.00	1.00	0.82	1.00	1.00	0.58	1.00	0.44
FINAL	1.00	1.00	1.00	1.00	1.00	0.95	1.00	1.00	1.00	0.52	1.00	0.32
HOUSE J24												
INITIAL	1.00	0.73	0.79	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	0.68	0.76	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J25												
INITIAL	1.00	1.00	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J26												
INITIAL	0.65	1.00	0.62	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11 (cont'd).

FINAL	0.56	1.00	0.38	0.90	0.99	1.00	1.00	1.00	1.00	0.84	1.00	1.00
HOUSE J27												
INITIAL	1.00	0.90	1.00	0.61	0.85	1.00	1.00	1.00	1.00	1.00	1.00	0.61
FINAL	1.00	0.38	1.00	0.25	0.37	1.00	1.00	1.00	1.00	1.00	1.00	0.39
HOUSE J28												
INITIAL	1.00	1.00	1.00	1.00	1.00	0.43	0.33	1.00	1.00	1.00	0.36	1.00
FINAL	1.00	1.00	1.00	1.00	1.00	0.39	0.31	1.00	1.00	1.00	0.39	1.00
HOUSE J29												
INITIAL	1.00	1.00	0.70	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.82
FINAL	1.00	1.00	0.64	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.81
HOUSE J30												
INITIAL	1.00	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	0.76	1.00	1.00
FINAL	1.00	1.00	1.00	1.00	1.00	0.94	1.00	1.00	1.00	0.69	1.00	1.00
HOUSE J32												
INITIAL	0.48	1.00	0.65	1.00	1.00	1.00	1.00	1.00	0.99	1.00	1.00	0.59
FINAL	0.35	1.00	0.64	1.00	1.00	1.00	1.00	1.00	0.84	1.00	1.00	0.41
HOUSE J33												
INITIAL	1.00	1.00	0.18	0.46	1.00	1.00	1.00	0.67	1.00	1.00	0.27	0.34
FINAL	1.00	1.00	0.18	0.41	1.00	1.00	1.00	0.75	1.00	1.00	0.33	0.42
HOUSE J34												
INITIAL	1.00	0.59	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.35
FINAL	0.60	0.44	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	1.00	0.22
HOUSE J38												
INITIAL	0.69	1.00	0.71	0.99	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00
FINAL	0.75	1.00	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.74	1.00
HOUSE J39												
INITIAL	1.00	0.88	1.00	1.00	0.45	0.39	1.00	1.00	1.00	1.00	0.94	1.00
FINAL	1.00	0.75	1.00	1.00	0.42	0.39	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J40												
INITIAL	1.00	1.00	0.71	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	0.68	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J41												
INITIAL	1.00	0.60	0.60	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	0.59	0.61	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J43												
INITIAL	0.66	1.00	0.30	0.52	1.00	1.00	1.00	1.00	1.00	1.00	0.64	1.00
FINAL	1.00	1.00	0.10	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.83	0.38
HOUSE J44												
INITIAL	1.00	1.00	0.86	0.48	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.93
FINAL	1.00	1.00	1.00	0.38	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J45												
INITIAL	1.00	0.66	1.00	0.90	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	0.28	1.00	0.49	1.00	1.00	0.71	1.00	1.00	1.00	1.00	1.00
HOUSE J46												
INITIAL	0.59	0.72	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.68	1.00
FINAL	0.49	0.79	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.59	1.00
HOUSE J47												
INITIAL	1.00	0.60	0.22	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.94	1.00
FINAL	1.00	1.00	0.14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.56
HOUSE J48												
INITIAL	1.00	1.00	1.00	0.70	1.00	1.00	1.00	1.00	1.00	1.00	0.48	1.00
FINAL	1.00	1.00	1.00	0.85	1.00	1.00	1.00	1.00	1.00	1.00	0.43	1.00
HOUSE J49												
INITIAL	0.74	1.00	1.00	0.87	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	0.61	1.00	1.00	0.76	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 11 (cont'd).

HOUSE J50												
INITIAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J51												
INITIAL	1.00	1.00	1.00	0.71	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.58
FINAL	1.00	1.00	1.00	0.78	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.59
HOUSE J54												
INITIAL	1.00	0.42	0.48	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	0.38	0.41	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J55												
INITIAL	0.81	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.87
FINAL	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.94
HOUSE J57												
INITIAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J58												
INITIAL	0.72	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.64
FINAL	0.66	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.59
HOUSE J60												
INITIAL	1.00	0.70	0.31	0.64	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	0.22	0.60	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J62												
INITIAL	1.00	1.00	1.00	0.69	0.96	1.00	0.67	1.00	1.00	1.00	1.00	0.47
FINAL	1.00	1.00	1.00	0.76	0.86	1.00	0.60	1.00	1.00	1.00	1.00	0.43
HOUSE J63												
INITIAL	0.90	1.00	0.52	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.36
FINAL	1.00	1.00	0.63	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.37
HOUSE J64												
INITIAL	1.00	0.83	0.54	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.90	1.00
FINAL	1.00	0.55	0.38	1.00	1.00	1.00	1.00	1.00	1.00	0.68	0.75	1.00
HOUSE J65												
INITIAL	0.99	1.00	0.88	1.00	1.00	1.00	1.00	0.87	1.00	1.00	0.72	1.00
FINAL	1.00	1.00	0.68	1.00	1.00	1.00	1.00	0.74	1.00	1.00	0.60	1.00
HOUSE J67												
INITIAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.69
FINAL	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.62
HOUSE J69												
INITIAL	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
HOUSE J70												
INITIAL	1.00	1.00	1.00	0.97	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
FINAL	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 12. Distribution of final and simulated weights for Elec-HO sample using RPRISM (W = weight). Simulated weights are defined as the weights which would be observed if the residuals had a normal distribution.

DISTRIBUTION OF SIMULATED WEIGHTS (200 IDEAL SAMPLES OF SIZE 12)

W	% OF ACTUAL WEIGHTS < W	% OF SIMULATED WEIGHTS < W
1	19.6	21.38
.95	18.4	19.38
.90	17.0	17.17
.85	15.6	15.58
.80	15.1	13.83
.75	13.9	11.58
.70	12.1	9.21
.60	8.5	5.92
.50	6.6	3.29
.40	4.5	1.42
.30	1.5	.25
.20	.87	.04

Table 13. By-house PRISM results for 50 electrically heated and cooled homes in New Jersey (Elec-AC sample). Units for PRISM parameters are as follows: τ(TREF) = °F; α(BASELEVEL) = kWh/day; β(SLOPE) = kWh/°F-day; NAC(Normalized Annual Consumption) = kWh/year; βH_o(HEATING PART) = kWh/year.

UNIT ID	TIME PERIOD	# PDS	# DAYS	RAW CONS	# ITS	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR	HEATING PART
J2810	12/13/78-12/13/79	12	365	20599	3	0.989	59.1(1.7)	27.68(1.88)	3.029(0.207)	21091.89(421.75)	10980.0(579.0)
J2815	12/28/78-12/28/79	12	365	22739	4	0.976	61.2(2.6)	24.13(3.88)	3.595(0.327)	23443.89(818.20)	14628.9(1188.3)
J2822	12/19/78-12/19/79	12	365	21329	3	0.925	67.5(6.1)	18.87(8.66)	2.680(0.404)	21729.59(1300.86)	14838.7(2758.6)
J2827	12/28/78-12/28/79	12	365	29839	5	0.952	58.0(3.3)	47.96(4.82)	3.820(0.509)	30534.96(1127.85)	13017.6(1430.4)
J2831	12/26/78-12/26/79	12	365	34469	4	0.983	58.3(2.0)	40.80(4.42)	5.912(0.474)	35349.95(1033.03)	20448.8(1307.2)
J2832	12/26/78-12/26/79	12	365	42259	4	0.833	59.3(7.1)	74.08(11.84)	4.395(1.206)	43136.22(2679.26)	16078.1(3599.8)
J2833	12/18/78-12/18/79	12	365	42909	3	0.957	57.7(3.2)	67.00(6.60)	5.717(0.747)	43638.31(1568.83)	19167.4(1951.7)
J2836	12/28/78-12/28/79	12	365	34109	3	0.917	52.8(4.4)	65.83(5.73)	4.310(0.862)	34660.31(1412.22)	10615.2(1674.3)
J2839	12/13/78-12/13/79	12	365	19929	4	0.951	58.0(3.9)	28.28(3.61)	2.939(0.466)	20327.00(848.85)	9996.5(1089.9)
J2855	12/13/78-12/13/79	12	365	23519	5	0.963	59.0(3.2)	29.18(4.32)	3.735(0.475)	24123.74(970.23)	13465.4(1332.0)
J2873	12/ 1/78-12/ 3/79	12	367	37519	3	0.977	59.1(2.4)	53.94(4.56)	5.020(0.465)	37921.39(1049.62)	18219.0(1380.2)
J2901	12/27/78-12/28/79	12	366	26669	7	0.881	65.0(7.2)	33.83(10.45)	3.012(0.619)	27174.64(1780.16)	14818.9(3334.8)
J2902	12/ 4/78-12/ 4/79	12	365	27559	5	0.913	60.0(4.7)	36.09(7.52)	3.945(0.713)	28211.73(1680.10)	15029.6(2253.8)
J2903	12/22/78-12/24/79	12	367	46749	2	0.983	62.7(2.3)	81.03(3.95)	3.970(0.310)	47006.64(787.82)	17411.7(1225.2)
J2906	12/22/78-12/24/79	12	367	21259	6	0.917	70.0(7.6)	19.36(10.51)	2.307(0.356)	21338.59(1247.32)	14266.5(3380.4)
J2923	12/ 5/78-12/ 5/79	12	365	16969	3	0.869	59.5(6.1)	18.85(6.35)	2.809(0.685)	17292.29(1483.21)	10407.6(1919.1)
J2929	12/13/78-12/13/79	12	365	14239	3	0.981	54.3(2.1)	16.32(2.04)	3.145(0.305)	14522.86(478.75)	8560.5(591.9)
J2933	12/ 5/78-12/ 5/79	12	365	33339	3	0.993	61.8(1.4)	32.20(2.93)	5.312(0.265)	34015.10(634.19)	22252.5(891.8)
J2937	12/18/78-12/18/79	12	365	32179	5	0.975	56.5(2.3)	47.94(4.12)	4.907(0.482)	32790.19(973.29)	15279.2(1202.8)
J2949	12/21/78-12/21/79	12	365	39559	5	0.926	54.0(3.8)	69.99(6.93)	5.414(0.927)	40017.87(1728.29)	14454.4(1947.8)
J2956	12/28/78-12/28/79	12	365	33029	3	0.970	63.3(3.1)	44.46(5.46)	3.901(0.388)	33866.09(1061.64)	17627.2(1698.1)
J2959	11/30/78-11/30/79	12	365	26989	3	0.983	63.2(2.3)	34.11(3.47)	3.362(0.245)	27628.53(679.01)	15169.4(1076.1)
J2975	12/15/78-12/17/79	12	367	35539	3	0.983	56.1(2.0)	52.06(3.74)	5.537(0.474)	35927.96(871.93)	16914.5(1104.5)
J2984	12/12/78-12/12/79	12	365	37239	3	0.989	55.5(1.7)	51.37(3.36)	6.488(0.498)	37842.44(795.38)	19078.9(994.9)
J2994	12/22/78-12/24/79	12	367	17869	4	0.874	56.0(5.0)	28.00(4.80)	2.558(0.570)	17971.90(1162.64)	7746.2(1377.3)
J2995	12/22/78-12/24/79	12	367	19039	3	0.995	61.8(1.3)	16.83(1.62)	3.138(0.139)	19316.95(337.56)	13168.4(492.6)
J3000	12/22/78-12/24/79	12	367	22549	3	0.981	59.8(2.2)	28.97(2.70)	3.254(0.278)	22819.61(617.84)	12239.1(814.4)
J3015	12/19/78-12/19/79	12	365	30759	6	0.751	41.3(7.6)	64.09(9.22)	7.356(3.672)	30545.85(2630.96)	7137.9(2550.6)
J3023	12/11/78-12/11/79	12	365	14589	3	0.991	61.9(1.7)	16.28(1.42)	2.117(0.129)	14872.33(295.38)	8924.8(437.2)
J3027	12/13/78-12/13/79	12	365	34119	3	0.988	61.1(1.9)	50.36(3.13)	4.063(0.278)	34828.97(645.25)	16436.1(964.3)
J3033	12/21/78-12/21/79	12	365	35699	3	0.882	52.0(4.6)	67.77(7.38)	4.847(1.066)	36090.84(1855.61)	11337.3(2108.6)
J3040	12/28/78-12/28/79	12	365	38689	3	0.912	54.9(4.3)	69.51(7.50)	4.942(0.933)	39411.52(1808.27)	14024.6(2120.6)
J3054	12/28/78-12/28/79	12	365	22089	4	0.979	58.5(2.2)	28.93(2.89)	3.461(0.305)	22680.07(676.30)	12115.1(857.7)
J3061	12/18/78-12/18/79	12	365	25339	3	0.986	54.4(1.6)	36.27(2.66)	4.597(0.337)	25813.52(635.12)	12566.7(753.5)
J3070	12/18/78-12/18/79	12	365	20309	3	0.985	56.8(1.8)	23.67(2.47)	3.812(0.289)	20784.57(583.79)	12137.6(721.5)
J3075	12/19/78-12/19/79	12	365	28599	5	0.823	67.0(9.6)	41.28(13.55)	2.611(0.631)	29200.22(2034.98)	14122.9(4315.4)
J3076	12/ 1/78-12/ 3/79	12	367	50679	5	0.819	50.1(7.3)	98.90(13.20)	7.174(2.610)	50835.78(3248.72)	14711.9(3891.6)
J3078	12/ 1/78-12/ 3/79	12	367	39719	4	0.971	56.3(2.6)	61.26(5.17)	5.749(0.623)	40113.20(1228.88)	17737.7(1523.0)
J3082	12/14/78-12/14/79	12	365	22709	3	0.975	63.7(2.9)	31.45(3.26)	2.529(0.237)	23191.51(625.82)	11703.1(1019.1)
J3087	11/30/78-11/30/79	12	365	38619	4	0.983	59.9(2.1)	55.14(4.18)	5.123(0.408)	39511.53(954.90)	19373.1(1273.2)
J3107	12/ 6/78-12/ 6/79	12	365	61059	3	0.980	63.0(2.5)	101.88(5.95)	5.528(0.461)	61857.50(1192.11)	24646.6(1845.8)
J3112	12/28/78-12/28/79	12	365	10749	3	0.731	60.4(9.6)	13.04(6.36)	1.626(0.576)	11095.67(1399.62)	6332.2(1923.4)
J3113	12/28/78-12/28/79	12	365	35749	3	0.931	55.1(3.8)	59.30(6.92)	5.166(0.861)	36483.77(1671.57)	14823.4(2025.6)
J3120	12/28/78-12/28/79	12	365	15699	3	0.987	61.5(1.9)	18.24(1.86)	2.299(0.157)	16159.68(391.68)	9497.0(568.9)
J3133	12/22/78-12/24/79	12	367	22559	3	0.974	54.3(2.1)	34.18(2.82)	3.748(0.358)	22705.92(688.97)	10220.9(788.8)
J3143	12/ 1/78-12/ 3/79	12	367	53959	4	0.969	59.3(2.8)	87.21(6.47)	6.133(0.659)	54352.79(1487.35)	22501.0(1955.8)
J3144	12/ 1/78-12/ 3/79	12	367	34069	4	0.989	51.4(1.7)	62.44(2.20)	5.067(0.412)	34145.24(540.39)	11339.2(671.5)
J3171	12/19/78-12/19/79	12	365	26269	4	0.989	61.0(1.7)	31.65(2.77)	3.809(0.233)	26869.25(577.71)	15310.6(848.0)
J3176	12/28/78-12/28/79	12	365	29269	4	0.983	58.3(2.0)	47.15(2.75)	3.633(0.290)	29846.89(642.90)	12623.9(815.3)
J3178	12/18/78-12/18/79	12	365	14069	5	0.844	49.0(5.5)	21.69(5.10)	3.322(0.950)	14187.59(1315.55)	6264.2(1439.7)

Table 14. By-house RPRISM results for Elec-AC sample. See Table 13 caption.

UNIT ID	TIME PERIOD	# PDS	# DAYS	RAW CONS	# ITS	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR	HEATING PART
J2810	12/13/78-12/13/79	12	365	20599	2	0.992	59.4(1.8)	26.79(1.88)	3.040(0.207)	20993.99(422.44)	11210.2(580.0)
J2815	12/28/78-12/28/79	12	365	22739	3	0.981	61.0(2.2)	24.19(3.25)	3.568(0.274)	23180.68(685.21)	14344.6(995.2)
J2822	12/19/78-12/19/79	12	365	21329	1	0.937	67.7(6.1)	18.60(8.74)	2.701(0.407)	21912.51(1311.74)	15117.5(2781.7)
J2827	12/28/78-12/28/79	12	365	29839	3	0.956	57.0(3.4)	48.32(4.96)	3.918(0.563)	30247.76(1188.21)	12597.3(1460.1)
J2831	12/26/78-12/26/79	12	365	34469	1	0.984	58.5(2.4)	41.19(5.25)	5.846(0.564)	35510.18(1227.77)	20464.6(1553.6)
J2832	12/26/78-12/26/79	12	365	42259	2	0.933	64.5(4.6)	64.47(8.42)	3.951(0.560)	42520.91(1563.86)	18972.4(2651.1)
J2833	12/18/78-12/18/79	12	365	42909	2	0.974	59.3(2.3)	67.69(4.68)	5.271(0.467)	44092.61(1053.37)	19367.8(1425.9)
J2836	12/28/78-12/28/79	12	365	34109	2	0.924	52.8(5.9)	66.38(7.59)	4.276(1.143)	34810.76(1872.14)	10564.5(2219.5)
J2839	12/13/78-12/13/79	12	365	19929	2	0.956	58.2(3.9)	27.76(3.99)	2.957(0.469)	20311.59(913.59)	10173.8(1208.3)
J2855	12/13/78-12/13/79	12	365	23519	3	0.975	60.0(2.6)	26.27(3.75)	3.707(0.362)	23719.52(812.71)	14123.9(1143.7)
J2873	12/ 1/78-12/ 3/79	12	367	37519	3	0.986	60.0(1.8)	50.78(3.71)	4.975(0.342)	37500.89(832.42)	18951.7(1109.7)
J2901	12/27/78-12/28/79	12	366	26669	3	0.882	65.0(7.8)	33.78(11.27)	3.023(0.668)	27212.65(1919.56)	14875.7(3596.0)
J2902	12/ 4/78-12/ 4/79	12	365	27559	2	0.964	58.4(1.8)	32.16(3.20)	4.878(0.354)	28759.86(747.91)	17012.6(958.6)
J2903	12/22/78-12/24/79	12	367	46749	2	0.986	63.3(2.2)	81.61(3.85)	3.855(0.280)	47289.38(744.31)	17482.9(1192.2)
J2906	12/22/78-12/24/79	12	367	21259	3	0.926	70.0(7.4)	19.52(10.34)	2.328(0.350)	21527.42(1227.98)	14398.9(3327.9)
J2923	12/ 5/78-12/ 5/79	12	365	16969	2	0.888	64.4(5.9)	16.83(5.67)	2.181(0.404)	16554.70(1080.54)	10406.8(1790.1)
J2929	12/13/78-12/13/79	12	365	14239	2	0.988	55.0(2.2)	14.69(2.06)	3.083(0.306)	14170.79(483.24)	8803.7(619.9)
J2933	12/ 5/78-12/ 5/79	12	365	33339	2	0.995	61.0(1.0)	32.52(2.29)	5.582(0.207)	34341.00(495.19)	22462.8(696.3)
J2937	12/18/78-12/18/79	12	365	32179	1	0.986	56.8(1.5)	45.39(2.71)	4.942(0.317)	32320.16(641.28)	15742.7(792.5)
J2949	12/21/78-12/21/79	12	365	39559	1	0.926	54.0(3.8)	69.99(6.93)	5.414(0.927)	40017.87(1728.23)	14454.4(1947.7)
J2956	12/28/78-12/28/79	12	365	33029	2	0.985	62.7(1.9)	43.00(3.24)	3.928(0.249)	32932.08(654.58)	17225.9(1008.1)
J2959	11/30/78-11/30/79	12	365	26989	2	0.987	63.7(2.5)	32.82(3.70)	3.351(0.261)	27501.09(724.65)	15512.4(1148.4)
J2975	12/15/78-12/17/79	12	367	35539	1	0.985	56.3(2.2)	52.50(4.05)	5.488(0.513)	36070.10(944.52)	16895.7(1196.5)
J2984	12/12/78-12/12/79	12	365	37239	1	0.991	55.9(1.7)	51.19(3.11)	6.283(0.461)	37612.08(735.72)	18914.4(920.2)
J2994	12/22/78-12/24/79	12	367	17869	2	0.901	54.4(4.4)	28.57(4.35)	2.697(0.552)	17825.59(1063.36)	7390.3(1217.4)
J2995	12/22/78-12/24/79	12	367	19039	1	0.995	61.9(1.3)	16.73(1.72)	3.136(0.148)	19304.27(359.47)	13193.6(524.5)
J3000	12/22/78-12/24/79	12	367	22549	1	0.983	59.5(2.3)	28.81(2.89)	3.280(0.297)	22695.40(660.57)	12172.2(870.7)
J3015	12/19/78-12/19/79	12	365	30759	1	0.768	41.5(7.8)	62.98(9.39)	7.374(3.737)	30260.70(2677.16)	7257.8(2595.4)
J3023	12/11/78-12/11/79	12	365	14589	2	0.993	62.1(1.2)	16.23(1.02)	2.087(0.085)	14833.18(206.34)	8905.3(318.6)
J3027	12/13/78-12/13/79	12	365	34119	1	0.991	60.8(1.7)	49.71(2.74)	4.140(0.264)	34631.42(593.19)	16474.6(834.7)
J3033	12/21/78-12/21/79	12	365	35699	3	0.884	52.0(4.9)	67.62(7.87)	4.839(1.138)	36017.16(1980.32)	11319.7(2250.3)
J3040	12/28/78-12/28/79	12	365	38689	2	0.944	55.5(4.3)	67.58(7.28)	4.948(0.905)	39205.90(1758.06)	14523.4(2130.4)
J3054	12/28/78-12/28/79	12	365	22089	1	0.985	59.1(1.8)	27.30(2.37)	3.439(0.238)	22451.25(545.60)	12479.7(717.0)
J3061	12/18/78-12/18/79	12	365	25339	1	0.994	55.2(1.1)	33.88(1.68)	4.512(0.213)	25421.12(401.48)	13046.3(492.7)
J3070	12/18/78-12/18/79	12	365	20309	1	0.991	57.9(1.3)	22.62(1.75)	3.653(0.198)	20647.53(415.58)	12386.6(517.0)
J3075	12/19/78-12/19/79	12	365	28599	2	0.853	64.8(7.8)	41.90(10.49)	2.900(0.691)	29408.99(1976.61)	14105.9(3305.1)
J3076	12/ 1/78-12/ 3/79	12	367	50679	2	0.945	53.4(3.3)	86.62(6.86)	6.714(1.004)	48937.80(1647.55)	17300.1(1999.9)
J3078	12/ 1/78-12/ 3/79	12	367	39719	2	0.977	56.6(2.0)	59.63(4.03)	5.714(0.485)	39696.73(957.04)	17916.6(1186.1)
J3082	12/14/78-12/14/79	12	365	22709	2	0.992	65.1(1.5)	30.87(1.74)	2.430(0.105)	23265.61(302.19)	11989.8(552.6)
J3087	11/30/78-11/30/79	12	365	38619	1	0.983	59.9(2.3)	55.12(4.51)	5.124(0.440)	39532.91(1029.93)	19400.7(1373.2)
J3107	12/ 6/78-12/ 6/79	12	365	61059	1	0.988	62.9(1.7)	99.22(4.03)	5.612(0.338)	61092.01(845.52)	24852.2(1243.8)
J3112	12/28/78-12/28/79	12	365	10749	2	0.841	58.7(5.7)	12.97(4.11)	1.918(0.434)	11545.25(963.08)	6808.1(1221.4)
J3113	12/28/78-12/28/79	12	365	35749	2	0.949	52.7(3.6)	60.24(6.07)	5.623(0.914)	35787.65(1497.29)	13786.2(1775.1)
J3120	12/28/78-12/28/79	12	365	15699	1	0.990	62.4(1.8)	18.01(1.70)	2.239(0.131)	16271.64(342.60)	9693.7(527.6)
J3133	12/22/78-12/24/79	12	367	22559	1	0.975	54.3(2.3)	34.15(3.13)	3.747(0.398)	22700.55(765.58)	10227.9(876.5)
J3143	12/ 1/78-12/ 3/79	12	367	53959	3	0.976	60.0(2.8)	88.01(7.01)	5.960(0.646)	54849.07(1570.79)	22704.6(2094.1)
J3144	12/ 1/78-12/ 3/79	12	367	34069	1	0.993	50.4(1.5)	62.32(1.95)	5.326(0.385)	33870.36(478.85)	11109.0(573.6)
J3171	12/19/78-12/19/79	12	365	26269	2	0.992	61.8(1.6)	31.77(2.43)	3.711(0.204)	27133.88(506.75)	15530.1(743.9)
J3176	12/28/78-12/28/79	12	365	29269	2	0.984	58.2(2.4)	47.66(3.32)	3.635(0.351)	29925.01(777.71)	12515.9(986.3)
J3178	12/18/78-12/18/79	12	365	14069	2	0.871	47.8(5.4)	20.71(4.16)	3.307(0.956)	13238.96(1122.69)	5673.9(1159.9)

Table 15. Summary statistics of PRISM vs. RPRISM parameters for Elec-AC sample.

	MEDIANS		MEANS		
	REGULAR	ROBUST	REGULAR	ROBUST	
ALPHA	38.54	37.67	44.22	43.19	kWh/day
BETA	3.86	3.88	4.14	4.13	kWh/°F-day
BHO	14195	14234	14035	14257	kWh/year
NAC	29524	29667	30186	30033	kWh/year
R-SQUARE	.975	.982	.941	.957	
CV ALPHA	.101	.096	.140	.126	
CV BETA	.098	.092	.140	.122	
CV BHO	.084	.073	.119	.104	
CV NAC	.030	.027	.040	.035	

Table 16. By-house percent differences for each estimate for Elec-AC sample. Percent difference is defined as:
[(RPRISM estimate-PRISM estimate)/PRISM estimate] * 100.

HOUSELET	PCALPHA	PCBETA	PCNAC	PCBHO
J2810	-3.215	0.363	-0.4642	2.0965
J2815	0.249	-0.751	-1.1227	-1.9434
J2822	-1.431	0.784	0.8418	1.8789
J2827	0.751	2.565	-0.9406	-3.2287
J2831	0.956	-1.116	0.4533	0.0773
J2832	-12.972	-10.102	-1.4264	18.0015
J2833	1.030	-7.801	1.0411	1.0455
J2836	0.835	-0.789	0.4341	-0.4776
J2839	-1.839	0.612	-0.0758	1.7736
J2855	-9.973	-0.750	-1.6756	4.8903
J2873	-5.858	-0.896	-1.1089	4.0216
J2901	-0.148	0.365	0.1399	0.3833
J2902	-10.889	23.650	1.9429	13.1940
J2903	0.716	-2.897	0.6015	0.4089
J2906	0.826	0.910	0.8849	0.9280
J2923	-10.716	-22.357	-4.2654	-0.0077
J2929	-9.988	-1.971	-2.4242	2.8410
J2933	0.994	5.083	0.9581	0.9451
J2937	-5.319	0.713	-1.4334	3.0335
J2949	0.000	0.000	0.0000	0.0000
J2956	-3.284	0.692	-2.7580	-2.2766
J2959	-3.782	-0.327	-0.4613	2.2611
J2975	0.845	-0.885	0.3956	-0.1111
J2984	-0.350	-3.160	-0.6087	-0.8622
J2994	2.036	5.434	-0.8141	-4.5945
J2995	-0.594	-0.064	-0.0656	0.1914
J3000	-0.552	0.799	-0.5443	-0.5466
J3015	-1.732	0.245	-0.9335	1.6798
J3023	-0.307	-1.417	-0.2632	-0.2185
J3027	-1.291	1.895	-0.5672	0.2342
J3033	-0.221	-0.165	-0.2042	-0.1552
J3040	-2.777	0.121	-0.5217	3.5566
J3054	-5.634	-0.636	-1.0089	3.0095
J3061	-6.589	-1.849	-1.5201	3.8164
J3070	-4.436	-4.171	-0.6593	2.0515
J3075	1.502	11.069	0.7150	-0.1204
J3076	-12.417	-6.412	-3.7336	17.5926
J3078	-2.661	-0.609	-1.0382	1.0086
J3082	-1.844	-3.915	0.3195	2.4498
J3087	-0.036	0.020	0.0541	0.1425
J3107	-2.611	1.520	-1.2375	0.8342
J3112	-0.537	17.958	4.0519	7.5156
J3113	1.585	8.846	-1.9080	-6.9970
J3120	-1.261	-2.610	0.6928	2.0712
J3133	-0.088	-0.027	-0.0237	0.0685
J3143	0.917	-2.821	0.9131	0.9048
J3144	-0.192	5.112	-0.8050	-2.0301
J3171	0.379	-2.573	0.9849	1.4336
J3176	1.082	0.055	0.2617	-0.8555
J3178	-4.518	-0.452	-6.6863	-9.4234

Table 17. Summary statistics of percent differences of PRISM vs. RPRISM parameters for Elec-AC sample. (See Figures 25-28.)

	MIN	Q1	MED	Q3	MAX	MEAN
ALPHA	-13.0	-3.9	-.57	.72	2.04	-2.3
BETA	-22.4	-1.9	-.11	.79	23.7	.146
NAC	-6.7	-1.1	-.46	.44	4.05	-.512
HEATING	-9.4	-.17	.86	2.3	18.0	1.45

Table 18. By-house summary of the ratio of the coefficients of variation (CVR) for PRISM vs. RPRISM estimates for Elec-AC sample. CVR = CV(PRISM estimate)/CV(RPRISM estimate).

HOUSELET	CVRALPHA	CVRBETA	CVRNAC	CVRBHO
J2810	0.96785	1.00363	0.99373	1.01921
J2815	1.19681	1.18447	1.18068	1.17083
J2822	0.97667	1.00041	1.00005	1.01033
J2827	0.97907	0.92728	0.94027	0.94803
J2831	0.84995	0.83104	0.84520	0.84205
J2832	1.22376	1.93601	1.68880	1.60229
J2833	1.42478	1.47478	1.50485	1.38306
J2836	0.76125	0.74821	0.75761	0.75076
J2839	0.88813	0.99969	0.92843	0.91801
J2855	1.03712	1.30232	1.17382	1.22160
J2873	1.15710	1.34746	1.24694	1.29378
J2901	0.92587	0.93003	0.92868	0.93092
J2902	2.09410	2.49047	2.29004	2.66135
J2903	1.03332	1.07507	1.06482	1.03188
J2906	1.02484	1.02640	1.02474	1.02520
J2923	0.99992	1.31648	1.31411	1.07198
J2929	0.89138	0.97708	0.96669	0.98196
J2933	1.29219	1.34526	1.29297	1.29287
J2937	1.43943	1.53135	1.49597	1.56377
J2949	1.00000	1.00000	1.00003	1.00005
J2956	1.62985	1.56902	1.57713	1.64611
J2959	0.90237	0.93563	0.93270	0.95823
J2975	0.93126	0.91580	0.92680	0.92208
J2984	1.07660	1.04613	1.07451	1.07186
J2994	1.12591	1.08872	1.08446	1.07937
J2995	0.93626	0.93859	0.93843	0.94098
J3000	0.92910	0.94351	0.93022	0.93023
J3015	0.96489	0.98501	0.97357	0.99925
J3023	1.38788	1.49614	1.42775	1.36926
J3027	1.12759	1.07299	1.08159	1.15797
J3033	0.93566	0.93519	0.93511	0.93558
J3040	1.00161	1.03219	1.02319	1.03080
J3054	1.15070	1.27337	1.22705	1.23223
J3061	1.47900	1.55290	1.55790	1.58769
J3070	1.34882	1.39872	1.39550	1.42418
J3075	1.31111	1.01424	1.03689	1.30411
J3076	1.68528	2.43291	1.89823	2.28823
J3078	1.24874	1.27672	1.27071	1.29699
J3082	1.83901	2.16878	2.07757	1.88937
J3087	0.92649	0.92745	0.92765	0.92850
J3107	1.43788	1.38463	1.39247	1.49638
J3112	1.53914	1.56553	1.51216	1.69310
J3113	1.15810	1.02535	1.09510	1.06127
J3120	1.08032	1.16720	1.15118	1.10061
J3133	0.90017	0.89926	0.89972	0.90056
J3143	0.93143	0.99135	0.95553	0.94241
J3144	1.12604	1.12483	1.11943	1.14691
J3171	1.14424	1.11277	1.15126	1.15628
J3176	0.83727	0.82667	0.82882	0.81955
J3178	1.17057	0.98924	1.09343	1.12426

Table 19. Summary statistics of CVRs of PRISM vs. RPRISM parameters for Elec-AC sample. (See Figures 30-33.)

	MIN	Q1	MED	Q3	MAX	MEAN
ALPHA	.76	.93	1.08	1.30	2.09	1.15
BETA	.75	.97	1.06	1.36	2.49	1.21
NAC	.76	.94	1.08	1.33	2.29	1.18
HEATING	.75	.95	1.08	1.32	2.66	1.20

Table 20. By-house PRISM results for 69 oil-heated homes in New Jersey (OIL). Units for PRISM parameters are as follows: τ(TREF) = °F; α(BASELEVEL) = gallons/day; β(SLOPE) = gallons/°F-day; NAC(Normalized Annual Consumption) = gallons/year; βH_0(HEATING PART) = gallons/year.

UNIT ID	TIME PERIOD	# PDS	# DAYS	RAW CONS	HW	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR	HEATING PART
P18646	11/11/76- 1/30/81	11	1541	1845	H	0.984	59.1(4.9)	0.04(0.19)	0.101(0.013)	384.11(22.77)	368.0(60.2)
P24939	12/22/78- 2/19/81	13	790	4573	HW	0.957	68.9(7.1)	0.64(1.29)	0.287(0.031)	1922.39(132.30)	1688.3(399.0)
P45584	1/19/78- 4/21/80	9	823	2950	HW	0.961	57.6(7.3)	1.33(0.59)	0.219(0.048)	1214.49(83.35)	728.0(171.2)
P56368	12/ 2/77- 1/22/81	11	1147	6853	HW	0.984	61.1(4.8)	1.76(0.69)	0.338(0.038)	2008.90(81.33)	1367.0(217.9)
P61454	1/25/79- 2/19/81	15	756	2508	HW	0.975	58.5(2.9)	0.79(0.37)	0.240(0.019)	1130.32(62.96)	842.8(93.3)
P64773	3/13/79- 4/30/80	9	414	3078	HW	0.970	62.7(4.4)	2.67(1.01)	0.415(0.050)	2795.51(144.89)	1820.1(264.8)
P69085	1/25/79- 2/ 5/81	13	742	7688	HW	0.966	69.3(6.5)	-1.34(2.95)	0.683(0.064)	3601.62(282.79)	4090.6(910.7)
P76263	10/12/79- 3/19/80	9	159	1462	HW	0.205	37.0(20.9)	8.98(0.97)	0.223(0.498)	3418.03(276.21)	137.1(118.3)
P88006	12/26/78- 1/20/81	12	756	1744	H	0.937	55.9(5.4)	0.33(0.57)	0.223(0.037)	790.92(102.59)	671.4(136.4)
P01249	12/12/77- 2/ 4/81	12	1150	4129	HW	0.993	62.9(2.3)	1.31(0.18)	0.169(0.009)	1230.78(26.57)	750.9(56.5)
P07232	1/19/79- 2/17/81	14	760	2371	H	0.972	80.0(26.3)	-1.96(4.40)	0.192(0.017)	1055.99(76.22)	1773.4(1618.2)
P07234	1/19/79- 2/17/81	14	760	2294	H	0.984	91.0(-9.9)	-3.78(-9.90)	0.183(0.007)	1028.41(52.09)	2409.2(-9.9)
P11122	11/24/80- 2/23/81	9	91	1675	HW	0.979	52.0(20.2)	6.71(9.91)	0.548(0.053)	3734.54(1902.14)	1281.9(1721.4)
P15920	10/24/78- 1/26/81	12	825	1650	H	0.881	61.9(11.2)	-0.23(0.98)	0.177(0.045)	660.09(126.56)	745.1(277.3)
P38343	2/14/79- 2/23/81	16	740	2971	HW	0.982	60.6(2.6)	1.31(0.33)	0.244(0.016)	1437.71(50.83)	960.3(87.0)
P45522	3/ 4/78- 1/27/81	12	1060	4088	HW	0.911	60.1(7.0)	1.58(0.77)	0.223(0.043)	1430.54(120.19)	852.4(209.0)
P53095	3/10/78- 2/ 4/81	12	1062	1984	H	0.979	64.4(5.0)	-0.33(0.47)	0.171(0.016)	702.26(52.06)	821.6(139.1)
P64672	12/ 1/78- 1/29/81	13	790	2263	H	0.957	72.1(10.1)	-0.66(1.25)	0.174(0.019)	940.73(86.14)	1181.1(413.2)
P65868	11/28/78- 1/17/81	12	781	2121	HW	0.938	64.6(6.5)	0.71(0.52)	0.139(0.020)	929.14(70.59)	669.5(146.6)
P75223	12/23/77- 4/16/79	9	479	3069	HW	0.830	74.3(32.3)	-0.89(7.40)	0.293(0.078)	1840.41(425.68)	2165.9(2469.0)
P12537	8/30/79- 2/ 3/81	14	523	1831	HW	0.952	69.5(8.3)	0.69(0.82)	0.151(0.017)	1170.09(71.41)	917.3(255.9)
P43660	12/22/77- 1/20/81	11	1125	3603	HW	0.980	67.3(6.6)	0.53(0.52)	0.164(0.021)	1094.84(45.34)	899.9(178.4)
P60414	2/ 3/79- 2/23/81	14	751	2091	H	0.970	59.6(3.7)	0.11(0.53)	0.245(0.021)	956.35(83.11)	915.0(133.1)
P67590	9/ 7/77- 3/14/80	9	919	3027	HW	0.988	63.3(3.6)	0.86(0.29)	0.171(0.016)	1086.17(38.09)	773.0(85.9)
P73573	3/17/78- 2/ 4/81	12	1055	2046	HW	0.990	72.2(5.5)	-0.12(0.41)	0.113(0.007)	724.90(24.89)	768.6(140.9)
P01990	2/13/79- 2/ 6/81	13	724	1923	H	0.954	56.8(4.4)	-0.04(0.70)	0.302(0.037)	948.04(116.77)	962.6(164.5)
P08723	4/ 9/77- 2/19/81	11	1412	3078	HW	0.936	61.5(11.0)	0.60(0.61)	0.139(0.034)	791.96(65.19)	572.3(205.0)
P34601	12/13/77- 1/23/81	11	1137	3772	HW	0.983	60.8(6.2)	0.68(0.54)	0.218(0.030)	1114.42(56.19)	866.6(175.4)
P36769	1/12/79- 5/ 5/80	9	479	3115	H	0.964	66.1(7.4)	0.28(1.99)	0.381(0.044)	2084.53(220.77)	1981.4(553.1)
P39089	1/ 3/78- 2/16/81	12	1140	3736	H	0.988	65.3(4.4)	-0.24(0.55)	0.234(0.019)	1077.70(55.29)	1167.1(173.1)
P40687	4/11/78- 1/20/81	11	1015	1709	HW	0.971	73.5(11.4)	-0.58(0.96)	0.122(0.014)	660.29(50.40)	873.7(330.3)
P54641	2/10/78- 2/13/81	12	1099	2125	H	0.960	66.0(9.8)	0.04(0.64)	0.129(0.023)	681.96(52.58)	665.8(211.0)
P63320	3/25/78- 2/ 6/81	12	1049	2088	HW	0.988	64.3(3.8)	0.02(0.30)	0.155(0.012)	747.62(32.50)	738.5(92.2)
P74200	12/ 9/80- 2/23/81	9	76	1614	H	0.983	50.0(4.9)	6.36(2.56)	0.689(0.051)	3724.60(504.67)	1400.1(443.8)
P96389	5/22/78- 1/19/81	12	973	3978	HW	0.979	65.4(3.5)	1.19(0.43)	0.224(0.018)	1560.50(62.23)	1124.8(130.6)
P96538	8/28/79- 2/12/81	16	534	2548	HW	0.976	71.4(5.7)	0.17(0.85)	0.226(0.017)	1547.11(68.04)	1484.8(279.6)
P97666	8/27/79- 2/13/81	17	536	2750	H	0.940	64.0(5.5)	0.16(1.23)	0.335(0.033)	1626.86(159.26)	1567.8(336.3)
P07303	2/20/79- 4/21/80	9	426	5377	HW	0.955	62.3(4.9)	4.54(1.95)	0.685(0.098)	4613.74(285.29)	2954.6(495.7)
P08396	12/21/77- 1/22/81	12	1128	3231	HW	0.941	65.0(12.8)	0.06(1.21)	0.190(0.041)	956.60(88.54)	933.8(404.1)
P23555	2/ 3/76- 4/ 4/80	9	1522	2757	H	0.955	47.6(5.6)	0.52(0.29)	0.248(0.069)	611.53(51.42)	421.5(81.9)
P55178	3/27/78- 2/20/81	13	1061	2274	H	0.970	62.3(4.3)	0.56(0.32)	0.137(0.013)	792.12(43.77)	588.3(90.1)
P74022	1/26/78- 1/17/81	12	1087	4671	HW	0.996	63.5(3.5)	1.04(0.41)	0.252(0.015)	1535.06(31.44)	1154.8(139.8)
P75517	1/12/78- 1/15/81	11	1099	1725	H	0.975	66.3(10.9)	-0.32(0.68)	0.127(0.022)	544.60(44.91)	662.6(236.3)
P02952	2/ 4/78- 1/ 9/81	11	1070	1774	HW	0.880	46.0(8.3)	0.97(0.28)	0.179(0.092)	620.38(58.80)	265.0(68.5)
P19896	10/13/76- 1/ 9/81	11	1549	3831	H	0.993	91.0(-9.9)	-3.52(-9.90)	0.159(0.005)	809.64(26.19)	2093.9(-9.9)
P37925	1/10/79- 2/20/81	14	772	2113	H	0.969	61.1(4.4)	-0.13(0.59)	0.232(0.021)	888.20(82.91)	937.2(158.9)
P57490	3/ 2/79- 2/ 6/81	14	707	2401	HW	0.965	76.5(13.0)	0.02(1.54)	0.158(0.016)	1285.04(58.86)	1277.6(550.4)
P81425	1/11/79- 2/21/81	14	772	2395	H	0.977	70.7(6.2)	-0.25(0.75)	0.177(0.013)	1035.79(62.21)	1127.9(235.0)
P83519	5/16/79- 5/ 1/80	9	351	1678	HW	0.976	61.8(3.6)	1.93(0.51)	0.257(0.027)	1783.94(80.64)	1080.2(128.6)
P83818	12/17/77- 1/23/81	11	1133	3730	HW	0.984	78.0(18.5)	-1.66(3.04)	0.199(0.017)	1096.90(65.92)	1704.4(1107.2)

Table 20 (cont'd).

P03401	2/ 5/79- 2/23/81	15	749	2320	HW	0.934	64.2(6.5)	1.15(0.49)	0.141(0.019)	1085.48(65.90)	666.0(147.4)
P18951	10/23/78- 2/ 3/81	12	834	1970	H	0.956	65.2(6.7)	-0.22(0.68)	0.169(0.021)	762.30(78.19)	841.6(197.5)
P19700	12/14/78- 2/13/81	13	792	1910	H	0.965	61.5(4.8)	-0.15(0.56)	0.198(0.020)	761.93(79.87)	815.6(147.7)
P19848	4/19/76- 1/ 9/81	10	1726	3409	HW	0.974	61.1(7.0)	0.44(0.36)	0.137(0.024)	712.67(36.57)	552.0(122.3)
P21045	3/ 4/78- 2/16/81	12	1080	2151	H	0.982	74.0(11.2)	-0.43(0.95)	0.121(0.012)	722.91(35.51)	881.6(332.3)
P31218	11/28/78- 5/30/80	9	549	3178	HW	0.957	58.8(8.0)	2.01(1.04)	0.321(0.073)	1877.72(137.89)	1142.1(286.6)
P35704	11/ 5/79- 2/20/81	18	473	3248	HW	0.916	68.4(6.9)	0.92(1.41)	0.301(0.035)	2073.55(162.46)	1739.3(417.7)
P39899	11/ 3/72- 9/14/73	9	315	1453	H	0.983	64.6(3.3)	-0.55(0.80)	0.403(0.035)	1742.72(109.48)	1942.1(206.0)
P48572	4/12/79- 2/19/81	15	679	4734	H	0.894	70.1(12.2)	-1.10(4.15)	0.494(0.078)	2669.30(353.08)	3070.1(1292.8)
P71306	12/10/77- 1/16/81	11	1133	3597	H	0.980	62.1(5.1)	0.16(0.59)	0.235(0.025)	1059.90(71.02)	1000.9(178.2)
P91342	3/ 8/79- 2/ 2/81	14	697	2391	H	0.950	66.9(5.8)	0.81(0.62)	0.188(0.022)	1307.80(75.67)	1011.9(187.5)
P03009	5/ 3/79- 2/16/81	16	655	2863	HW	0.989	68.7(2.7)	0.17(0.49)	0.270(0.011)	1641.41(53.96)	1579.7(146.8)
P05292	5/12/76- 9/ 2/80	10	1574	2023	HW	0.287	58.0(55.9)	2.14(0.56)	-0.078(0.218)	517.35(140.17)	-264.8(207.8)
P13631	4/14/76- 1/17/81	10	1739	3136	H	0.990	56.4(4.2)	0.11(0.22)	0.193(0.025)	637.58(27.30)	597.2(78.3)
P40400	5/ 1/78- 1/27/81	12	1002	2470	HW	0.989	67.3(6.2)	-0.07(0.55)	0.179(0.018)	951.36(38.44)	978.1(188.6)
P48878	5/17/77- 1/23/81	11	1347	3870	HW	0.972	65.5(5.7)	-0.19(0.68)	0.223(0.023)	1052.12(76.09)	1122.9(218.5)
P78349	3/22/78- 5/ 6/80	9	776	4730	HW	0.909	89.0(-9.9)	-4.72(-9.90)	0.317(0.038)	2226.38(215.23)	3952.0(-9.9)
P02626	1/15/79- 1/29/81	13	745	2031	H	0.977	63.8(4.2)	-0.37(0.61)	0.231(0.018)	939.12(80.05)	1073.0(166.3)
P05616	3/ 5/79- 2/11/81	15	709	2685	H	0.986	75.0(6.8)	-0.77(1.17)	0.223(0.012)	1416.92(54.78)	1698.4(405.6)

Table 21. By-house OUTWTS results for OIL sample. See Table 20 caption.

UNIT ID	TIME PERIOD	# PDS	# DAYS	RAW CONS	HW	RXR	TREF	BASELEVEL X PER DAY	SLOPE X PER HDD	NAC X PER YEAR	HEATING PART
P18646	11/11/76- 1/30/81	11	1541	1845	H	0.987	57.8(5.0)	0.08(0.14)	0.105(0.016)	383.85(15.39)	353.5(50.9)
P24939	12/22/78- 2/19/81	13	790	4573	HW	0.973	69.8(5.5)	0.52(0.80)	0.281(0.028)	1914.32(83.14)	1726.0(278.1)
P45584	1/19/78- 4/21/80	9	823	2950	HW	0.969	54.5(5.4)	1.51(0.36)	0.242(0.051)	1220.25(57.38)	667.9(110.1)
P56368	12/ 2/77- 1/22/81	11	1147	6853	HW	0.989	63.7(4.0)	1.45(0.49)	0.321(0.030)	2011.99(55.06)	1482.1(172.6)
P61454	1/25/79- 2/19/81	15	756	2508	HW	0.983	58.1(2.1)	0.82(0.18)	0.241(0.018)	1125.97(39.05)	826.6(56.6)
P64773	3/13/79- 4/30/80	9	414	3078	HW	0.978	60.7(3.1)	2.95(0.52)	0.438(0.054)	2813.85(102.23)	1736.5(148.8)
P69085	1/25/79- 2/ 5/81	13	742	7688	HW	0.985	70.6(4.9)	-1.72(1.69)	0.665(0.056)	3599.00(146.76)	4227.9(594.9)
P76263	10/12/79- 3/19/80	9	159	1462	HW	0.655	75.0(-9.9)	4.43(-9.90)	0.140(0.039)	2687.69(222.80)	1069.8(-9.9)
P88006	12/26/78- 1/20/81	12	756	1744	H	0.952	55.2(5.3)	0.27(0.35)	0.229(0.044)	758.78(65.85)	659.9(115.2)
P01249	12/12/77- 2/ 4/81	12	1150	4129	HW	0.990	63.3(2.4)	1.23(0.17)	0.170(0.011)	1219.95(24.91)	770.6(57.0)
P07232	1/19/79- 2/17/81	14	760	2371	H	0.989	91.0(-9.9)	-3.91(-9.90)	0.188(0.006)	1056.48(34.43)	2485.1(-9.9)
P07234	1/19/79- 2/17/81	14	760	2294	H	0.994	91.0(-9.9)	-3.76(-9.90)	0.182(0.004)	1022.50(25.56)	2397.6(-9.9)
P11122	11/24/80- 2/23/81	9	91	1675	HW	0.979	52.0(22.7)	6.73(10.98)	0.546(0.059)	3736.19(2095.06)	1278.0(1917.6)
P15920	10/24/78- 1/26/81	12	825	1650	H	0.951	59.8(6.3)	-0.12(0.39)	0.182(0.034)	640.46(56.27)	685.2(127.9)
P38343	2/14/79- 2/23/81	16	740	2971	HW	0.988	61.6(2.4)	1.24(0.20)	0.237(0.017)	1434.24(32.19)	982.8(65.9)
P45522	3/ 4/78- 1/27/81	12	1060	4088	HW	0.941	59.8(5.6)	1.56(0.40)	0.229(0.046)	1431.05(74.05)	861.6(128.6)
P53095	3/10/78- 2/ 4/81	12	1062	1984	H	0.988	66.2(5.1)	-0.44(0.37)	0.165(0.017)	700.50(27.31)	861.3(134.0)
P64672	12/ 1/78- 1/29/81	13	790	2263	H	0.978	70.3(5.6)	-0.50(0.55)	0.178(0.015)	933.27(43.72)	1114.3(193.7)
P65868	11/28/78- 1/17/81	12	781	2121	HW	0.963	64.8(5.0)	0.69(0.28)	0.138(0.019)	922.80(44.05)	671.3(92.5)
P75223	12/23/77- 4/16/79	9	479	3069	HW	0.903	76.1(32.9)	-1.16(6.86)	0.278(0.070)	1794.11(239.88)	2216.1(2449.2)
P12537	8/30/79- 2/ 3/81	14	523	1831	HW	0.975	70.7(6.5)	0.61(0.52)	0.147(0.015)	1163.89(38.97)	940.0(182.8)
P43660	12/22/77- 1/20/81	11	1125	3603	HW	0.984	66.7(6.0)	0.58(0.40)	0.166(0.021)	1097.64(35.49)	886.0(144.4)
P60414	2/ 3/79- 2/23/81	14	751	2091	H	0.982	59.5(2.9)	0.08(0.25)	0.247(0.022)	942.96(44.23)	914.2(83.5)
P67590	9/ 7/77- 3/14/80	9	919	3027	HW	0.991	63.2(3.2)	0.87(0.21)	0.171(0.016)	1086.27(27.94)	769.0(66.4)
P73573	3/17/78- 2/ 4/81	12	1055	2046	HW	0.994	70.8(3.3)	-0.04(0.21)	0.116(0.006)	724.20(14.55)	740.1(72.9)
P01990	2/13/79- 2/ 6/81	13	724	1923	H	0.977	56.6(3.1)	-0.01(0.28)	0.301(0.034)	945.25(53.85)	948.7(90.4)
P08723	4/ 9/77- 2/19/81	11	1412	3078	HW	0.937	63.1(11.2)	0.52(0.56)	0.133(0.034)	786.74(49.11)	597.4(206.3)
P34601	12/13/77- 1/23/81	11	1137	3772	HW	0.986	58.7(5.1)	0.84(0.35)	0.227(0.032)	1114.39(36.87)	805.9(127.2)
P36769	1/12/79- 5/ 5/80	9	479	3115	H	0.976	64.8(5.1)	0.44(0.91)	0.394(0.051)	2080.44(122.05)	1919.3(282.7)
P39089	1/ 3/78- 2/16/81	12	1140	3736	H	0.994	66.6(4.2)	-0.36(0.42)	0.228(0.018)	1076.63(29.86)	1209.5(152.9)
P40687	4/11/78- 1/20/81	11	1015	1709	HW	0.981	71.4(7.5)	-0.44(0.51)	0.124(0.014)	654.26(29.96)	815.1(181.9)
P54641	2/10/78- 2/13/81	12	1099	2125	H	0.980	67.0(9.4)	-0.01(0.53)	0.127(0.021)	680.81(28.74)	685.0(192.6)
P63320	3/25/78- 2/ 6/81	12	1049	2088	HW	0.993	62.8(3.1)	0.10(0.18)	0.161(0.013)	745.90(19.59)	710.6(61.8)
P74200	12/ 9/80- 2/23/81	9	76	1614	H	0.986	50.0(4.4)	6.20(2.23)	0.698(0.049)	3681.67(432.06)	1417.1(396.3)
P96389	5/22/78- 1/19/81	12	973	3978	HW	0.982	65.6(2.9)	1.16(0.28)	0.223(0.018)	1552.16(47.64)	1128.2(95.6)
P96538	8/28/79- 2/12/81	16	534	2548	HW	0.985	70.7(4.0)	0.25(0.47)	0.227(0.016)	1539.71(44.94)	1450.1(165.5)
P97666	8/27/79- 2/13/81	17	536	2750	H	0.968	63.8(3.5)	0.10(0.50)	0.338(0.031)	1602.16(81.68)	1563.9(162.2)
P07303	2/20/79- 4/21/80	9	426	5377	HW	0.976	64.3(3.9)	4.04(1.07)	0.656(0.086)	4594.11(174.79)	3118.2(319.9)
P08396	12/21/77- 1/22/81	12	1128	3231	HW	0.957	63.7(10.1)	0.21(0.75)	0.193(0.041)	963.17(56.58)	887.5(273.4)
P23555	2/ 3/76- 4/ 4/80	9	1522	2757	H	0.954	66.0(10.1)	-0.28(0.59)	0.138(0.031)	608.62(40.61)	710.0(209.1)
P55178	3/27/78- 2/20/81	13	1061	2274	H	0.982	62.7(3.4)	0.53(0.17)	0.136(0.013)	790.67(25.44)	596.7(58.6)
P74022	1/26/78- 1/17/81	12	1087	4671	HW	0.996	63.4(3.4)	1.05(0.36)	0.252(0.016)	1533.10(23.13)	1150.7(130.1)
P75517	1/12/78- 1/15/81	11	1099	1725	H	0.983	64.6(9.1)	-0.23(0.48)	0.129(0.023)	541.70(26.75)	624.7(174.1)
P02952	2/ 4/78- 1/ 9/81	11	1070	1774	HW	0.887	58.8(11.3)	0.70(0.29)	0.099(0.040)	607.89(45.73)	351.1(97.1)
P19896	10/13/76- 1/ 9/81	11	1549	3831	H	0.991	91.0(-9.9)	-3.48(-9.90)	0.158(0.005)	812.69(21.32)	2083.4(-9.9)
P37925	1/10/79- 2/20/81	14	772	2113	H	0.985	61.5(3.2)	-0.19(0.27)	0.231(0.019)	882.28(39.05)	950.0(93.6)
P57490	3/ 2/79- 2/ 6/81	14	707	2401	HW	0.973	72.0(6.5)	0.45(0.57)	0.164(0.017)	1268.69(43.72)	1104.1(203.8)
P81425	1/11/79- 2/21/81	14	772	2395	H	0.990	69.9(4.1)	-0.20(0.38)	0.180(0.012)	1034.80(30.15)	1108.7(132.9)
P83519	5/16/79- 5/ 1/80	9	351	1678	HW	0.983	62.7(3.0)	1.80(0.29)	0.253(0.028)	1771.14(57.39)	1112.5(87.7)
P83818	12/17/77- 1/23/81	11	1133	3730	HW	0.988	70.6(6.2)	-0.68(0.73)	0.212(0.018)	1096.43(40.95)	1345.1(260.5)

Table 21 (cont'd).

P03401	2/ 5/79- 2/23/81	15	749	2320	HW	0.967	61.8(4.1)	1.22(0.22)	0.152(0.018)	1087.53(37.92)	640.9(74.7)
P18951	10/23/78- 2/ 3/81	12	834	1970	H	0.980	67.4(6.6)	-0.35(0.50)	0.161(0.020)	757.84(37.86)	885.7(172.6)
P19700	12/14/78- 2/13/81	13	792	1910	H	0.981	62.5(4.3)	-0.24(0.31)	0.193(0.020)	752.24(40.76)	840.0(109.5)
P19848	4/19/76- 1/ 9/81	10	1726	3409	HW	0.976	59.6(6.9)	0.49(0.29)	0.144(0.028)	716.17(27.94)	537.2(106.8)
P21045	3/ 4/78- 2/16/81	12	1080	2151	H	0.990	71.8(6.6)	-0.28(0.44)	0.123(0.011)	721.42(20.35)	825.4(156.7)
P31218	11/28/78- 5/30/80	9	549	3178	HW	0.965	55.5(5.8)	2.33(0.61)	0.354(0.078)	1894.17(96.52)	1041.8(177.3)
P35704	11/ 5/79- 2/20/81	18	473	3248	HW	0.957	65.7(4.1)	1.13(0.59)	0.321(0.034)	2042.51(96.68)	1629.4(191.5)
P39899	11/ 3/72- 9/14/73	9	315	1453	H	0.993	63.4(2.5)	-0.46(0.34)	0.418(0.037)	1733.29(57.75)	1900.7(107.8)
P48572	4/12/79- 2/19/81	15	679	4734	H	0.956	68.4(6.8)	-0.75(1.63)	0.508(0.069)	2647.06(165.69)	2922.1(581.5)
P71306	12/10/77- 1/16/81	11	1133	3597	H	0.984	63.2(5.1)	0.06(0.47)	0.230(0.026)	1057.52(46.96)	1034.7(165.7)
P91342	3/ 8/79- 2/ 2/81	14	697	2391	H	0.956	66.4(4.9)	0.83(0.41)	0.190(0.024)	1299.56(60.02)	997.8(138.8)
P03009	5/ 3/79- 2/16/81	16	655	2863	HW	0.995	69.2(1.8)	0.11(0.24)	0.268(0.010)	1644.08(29.57)	1603.0(85.1)
P05292	5/12/76- 9/ 2/80	10	1574	2023	HW	0.264	85.0(****)	2.42(35.58)	-0.038(0.032)	461.83(131.93)	-420.4(12955.3)
P13631	4/14/76- 1/17/81	10	1739	3136	H	0.986	56.5(4.7)	0.13(0.22)	0.190(0.029)	639.73(24.12)	593.4(85.1)
P40400	5/ 1/78- 1/27/81	12	1002	2470	HW	0.993	70.3(6.1)	-0.34(0.54)	0.171(0.015)	946.92(24.26)	1071.8(195.6)
P48878	5/17/77- 1/23/81	11	1347	3870	HW	0.974	67.7(5.7)	-0.35(0.55)	0.212(0.024)	1053.88(51.35)	1183.5(202.2)
P78349	3/22/78- 5/ 6/80	9	776	4730	HW	0.945	89.0(-9.9)	-5.19(-9.90)	0.329(0.030)	2210.05(153.26)	4106.3(-9.9)
P02626	1/15/79- 1/29/81	13	745	2031	H	0.988	62.8(3.2)	-0.28(0.28)	0.234(0.019)	930.07(38.79)	1031.6(98.5)
P05616	3/ 5/79- 2/11/81	15	709	2685	H	0.992	74.3(4.9)	-0.67(0.73)	0.225(0.012)	1413.31(30.70)	1659.8(264.6)

Table 22. Summary statistics of PRISM vs. OUTWTS parameters for OIL sample.

	MEDIANS		MEANS		
	REGULAR	OUTWTS	REGULAR	OUTWTS	
ALPHA	.16	.13	.475	.381	gal/day
BETA	.219	.212	.239	.237	gal/°F-day
BHO	963	998	1197	1215	gal/year
NAC	1086	1086	1371	1354	gal/year
R-SQUARE	.971	.982	.942	.961	
CV ALPHA	.393	.241	1.53	-.953	
	(HW=.487)	(HW=.410)	(HW=4.40)	(HW=1.22)	
CV BETA	.106	.104	.113	.107	
CV BHO	.198	.141	.247	-.274	
CV NAC	.068	.041	.081	.055	

Table 23. By-house percent differences for each estimate for OIL sample. Percent difference is defined as:
[(RPRISM estimate-PRISM estimate)/PRISM estimate] * 100.

HOUSELET	PCALPHA	PCBETA	PCNAC	PCBHO	HOT WATER
P18646	100.00	3.960	-0.068	-3.940	H
P24939	-18.75	-2.091	-0.420	2.233	HW
P45584	13.53	10.502	0.474	-8.255	HW
P56368	-17.61	-5.030	0.154	8.420	HW
P61454	3.80	0.417	-0.385	-1.922	HW
P64773	10.49	5.542	0.656	-4.593	HW
P69085	28.36	-2.635	-0.073	3.356	HW
P76263	-50.67	-37.220	-21.367	680.306	HW
P88006	-18.18	2.691	-4.064	-1.713	H
P01249	-6.11	0.592	-0.880	2.624	HW
P07232	99.49	-2.083	0.046	40.132	H
P07234	-0.53	-0.546	-0.575	-0.481	H
P11122	0.30	-0.365	0.044	-0.304	HW
P15920	-47.83	2.825	-2.974	-8.039	H
P38343	-5.34	-2.869	-0.241	2.343	HW
P45522	-1.27	2.691	0.036	1.079	HW
P53095	33.33	-3.509	-0.251	4.832	H
P64672	-24.24	2.299	-0.793	-5.656	H
P65868	-2.82	-0.719	-0.682	0.269	HW
P75223	30.34	-5.119	-2.516	2.318	HW
P12537	-11.59	-2.649	-0.530	2.475	HW
P43660	9.43	1.220	0.256	-1.545	HW
P60414	-27.27	0.816	-1.400	-0.087	H
P67590	1.16	0.000	0.009	-0.517	HW
P73573	-66.67	2.655	-0.097	-3.708	HW
P01990	-75.00	-0.331	-0.294	-1.444	H
P08723	-13.33	-4.317	-0.659	4.386	HW
P34601	23.53	4.128	-0.003	-7.004	HW
P36769	57.14	3.412	-0.196	-3.134	H
P39089	50.00	-2.564	-0.099	3.633	H
P40687	-24.14	1.639	-0.913	-6.707	HW
P54641	-125.00	-1.550	-0.169	2.884	H
P63320	400.00	3.871	-0.230	-3.778	HW
P74200	-2.52	1.306	-1.153	1.214	H
P96389	-2.52	-0.446	-0.534	0.302	HW
P96538	47.06	0.442	-0.478	-2.337	HW
P97666	-37.50	0.896	-1.518	-0.249	H
P07303	-11.01	-4.234	-0.425	5.537	HW
P08396	250.00	1.579	0.687	-4.958	HW
P23555	-153.85	-44.355	-0.476	68.446	H
P55178	-5.36	-0.730	-0.183	1.428	H
P74022	0.96	0.000	-0.128	-0.355	HW
P75517	-28.12	1.575	-0.533	-5.720	H
P02952	-27.84	-44.693	-2.013	32.491	HW
P19896	-1.14	-0.629	0.377	-0.501	H
P37925	46.15	-0.431	-0.667	1.366	H
P57490	2150.00	3.797	-1.272	-13.580	HW
P81425	-20.00	1.695	-0.096	-1.702	H
P83519	-6.74	-1.556	-0.718	2.990	HW
P83818	-59.04	6.533	-0.043	-21.081	HW
P03401	6.09	7.801	0.189	-3.769	HW
P18951	59.09	-4.734	-0.585	5.240	H
P19700	60.00	-2.525	-1.272	2.992	H
P19848	11.36	5.109	0.491	-2.681	HW

Table 23 (cont'd).

HOUSELET	PCALPHA	PCBETA	PCNAC	PCBHO	HOT WATER
P21045	-34.884	1.653	-0.206	-6.3748	H
P31218	15.920	10.280	0.876	-8.7821	HW
P35704	22.826	6.645	-1.497	-6.3186	HW
P39899	-16.364	3.722	-0.541	-2.1317	H
P48572	-31.818	2.834	-0.833	-4.8207	H
P71306	-62.500	-2.128	-0.225	3.3770	H
P91342	2.469	1.064	-0.630	-1.3934	H
P03009	-35.294	-0.741	0.163	1.4750	HW
P05292	13.084	-51.282	-10.732	58.7613	HW
P13631	18.182	-1.554	0.337	-0.6363	H
P40400	385.714	-4.469	-0.467	9.5798	HW
P48878	84.211	-4.933	0.167	5.3967	HW
P78349	9.958	3.785	-0.733	3.9044	HW
P02626	-24.324	1.299	-0.964	-3.8583	H
P05616	-12.987	0.897	-0.255	-2.2727	H

Table 24. Summary statistics of percent differences of PRISM vs. RPRISM parameters for OIL sample. (See Figures 50-53.)

	MIN	Q1	MED	Q3	MAX	MEAN
ALPHA	-154	-24.2	-1.3	23.2	2150	42.5
BETA	-51.3	-2.3	.42	2.7	10.5	-1.9
NAC	-21.4	-.76	-.38	-.023	.876	-.92
HEATING	-21	-3.17	-.35	2.99	680	11.73

Table 25. By-house summary of the ratio of the coefficients of variation (CVR) for PRISM vs. RPRISM estimates for OIL sample. CVR = CV(PRISM estimate)/CV(RPRISM estimate).

HOUSELET	CVRALPHA	CVRBETA	CVRNAC	CVRBHO	HOT WATER
P18646	2.7143	0.84468	1.47853	1.13611	H
P24939	1.3102	1.08400	1.58461	1.46677	HW
P45584	1.8607	1.04002	1.45949	1.42658	HW
P56368	1.1601	1.20296	1.47939	1.36875	HW
P61454	2.1336	1.05995	1.60609	1.61672	HW
P64773	2.1460	0.97724	1.42659	1.69783	HW
P69085	2.2406	1.11274	1.92549	1.58223	HW
P76263	-0.0483	8.01656	0.97483	.	HW
P88006	1.3325	0.86353	1.49463	1.16375	H
P01249	0.9942	0.82302	1.05725	1.01723	HW
P07232	-0.8866	2.77431	2.21479	.	H
P07234	0.9947	1.74044	2.02624	.	H
P11122	0.9052	0.89503	0.90832	0.89495	HW
P15920	1.3110	1.36092	2.18227	1.99380	H
P38343	1.5618	0.91418	1.57525	1.35111	HW
P45522	1.9006	0.95993	1.62367	1.64274	HW
P53095	1.6937	0.90815	1.90148	1.08822	H
P64672	1.7218	1.29579	1.95464	2.01255	H
P65868	1.8048	1.04506	1.59156	1.58913	HW
P75223	1.4060	1.05724	1.72991	1.03145	HW
P12537	1.3941	1.10331	1.82273	1.43453	HW
P43660	1.4226	1.01220	1.28081	1.21637	HW
P60414	1.5418	0.96234	1.85273	1.59262	H
P67590	1.3970	1.00000	1.36340	1.28698	HW
P73573	0.6508	1.19764	1.70900	1.86112	HW
P01990	0.6250	1.08463	2.16205	1.79341	H
P08723	0.9440	0.95683	1.31868	1.03728	HW
P34601	1.9059	0.97620	1.52396	1.28235	HW
P36769	3.4364	0.89218	1.80530	1.89517	H
P39089	1.9643	1.02849	1.84980	1.17324	H
P40687	1.4280	1.01639	1.66688	1.69404	HW
P54641	-0.3019	1.07826	1.82642	1.12713	H
P63320	8.3333	0.95881	1.65519	1.43555	HW
P74200	1.1191	1.05441	1.15459	1.13346	H
P96389	1.4970	0.99554	1.29927	1.37024	HW
P96538	2.6596	1.06720	1.50678	1.64994	HW
P97666	1.5375	1.07405	1.92020	2.06821	H
P07303	1.6217	1.09129	1.62524	1.63535	HW
P08396	5.6467	1.01579	1.57561	1.40477	HW
P23555	-0.2647	1.23855	1.26017	0.65977	H
P55178	1.7815	0.99270	1.71737	1.55950	H
P74022	1.1498	0.93750	1.35754	1.07074	HW
P75517	1.0182	0.97159	1.66994	1.27963	H
P02952	0.6968	1.27207	1.25992	0.93467	HW
P19896	0.9886	0.99371	1.23305	.	H
P37925	3.1937	1.10050	2.10902	1.72084	H
P57490	60.7895	0.97692	1.32917	2.33393	HW
P81425	1.5789	1.10169	2.06138	1.73815	H
P83519	1.6402	0.94928	1.39504	1.51021	HW
P83818	1.7059	1.00614	1.60908	3.35430	HW
P03401	2.3628	1.13790	1.74115	1.89886	HW
P18951	2.1636	1.00030	2.05316	1.20422	H
P19700	2.8903	0.97475	1.93460	1.38921	H
P19848	1.3824	0.90094	1.31530	1.11443	HW

Table 25 (cont'd).

HOUSELET	CVRALPHA	CVRBETA	CVRNAC	CVRBHO	HOT WATER
P21045	1.40592	1.10894	1.74137	1.98543	H
P31218	1.97635	1.03211	1.44113	1.47451	HW
P35704	2.93534	1.09781	1.65523	2.04338	HW
P39899	1.96791	0.98115	1.88550	1.87021	H
P48572	1.73592	1.16247	2.11321	2.11604	H
P71306	0.47074	0.94108	1.50895	1.11175	H
P91342	1.54953	0.92642	1.25280	1.33204	H
P03009	1.32108	1.09185	1.82779	1.75047	HW
P05292	0.01780	3.31891	0.94844	0.02546	HW
P13631	1.18182	0.84867	1.13566	0.91424	H
P40400	4.94709	1.14637	1.57711	1.05658	HW
P48878	2.27751	0.91106	1.48427	1.13893	HW
P78349	1.09958	1.31462	1.39404	.	HW
P02626	1.64865	0.95967	2.04379	1.62318	H
P05616	1.39459	1.00897	1.77982	1.49804	H

Table 26. Summary statistics of CVRs of PRISM vs. RPRISM parameters for OIL sample. (See Figures 55-58.)

	MIN	Q1	MED	Q3	MAX	MEAN
ALPHA	.018	.99	1.46	1.96	60.8	3.76
BETA	.82	.96	1.02	1.10	8.02	1.20
NAC	.91	1.38	1.61	1.84	2.21	1.61
HEATING	.025	1.14	1.44	1.72	3.35	1.47

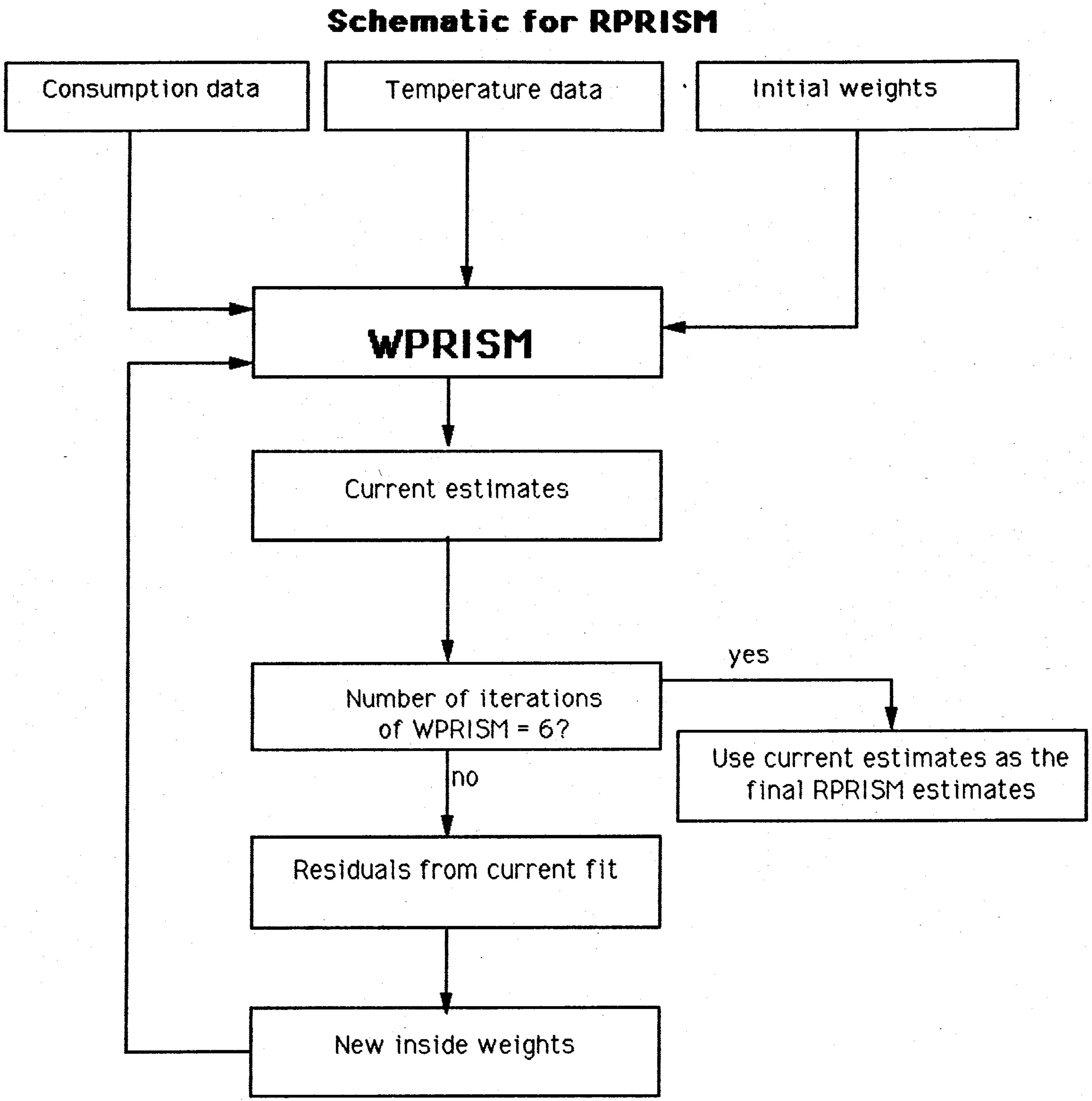


Figure 1. Schematic of Robust PRISM (RPRISM) algorithm.

```
VARIABLE=PCALPHA

              MOMENTS                                     QUANTILES(DEF=4)                     EXTREMES

N                 50  SUM WGTS          50  100% MAX    171.248      99%     171.248      LOWEST     HIGHEST
MEAN         3.00927  SUM          150.463   75% Q3    0.825371      95%      5.9828    -14.6711     3.56875
STD DEV      24.5589  VARIANCE     603.139   50% MED  0.0859245      90%     3.54484    -11.6583     4.04722
SKEWNESS     6.81459  KURTOSIS     47.6208   25% Q1   -0.280762      10%    -3.69776    -10.0389     5.30973
USS          30006.6  CSS          29553.8    0% MIN   -14.6711       5%    -10.7676    -7.09505     6.80544
CV           816.109  STD MEAN     3.47315                            1%    -14.6711    -3.70634     171.248
T:MEAN=0    0.866437  PROB>|T|    0.390473  RANGE     185.919
SGN RANK          95  PROB>|S|    0.317304  Q3-Q1     1.10613
NUM ¬= 0          47                        MODE            0

STEM LEAF                              #  BOXPLOT                        NORMAL PROBABILITY PLOT
  17 1                                 1     *         175+                                                  *
  16                                                      |
  15                                                      |
  14                                                      |
  13                                                      |
  12                                                      |
  11                                                      |
  10                                                      |
   9                                                      |
   8                                                      |
   7                                                      |
   6                                                      |                                                    ++
   5                                                      |                                                ++++
   4                                                      |                                            ++++
   3                                                      |                                       +++++
   2                                                      |                                   ++++
   1                                                      |                               ++++
   0 1111111122234457                 16  *--0--*         |                     ***************** ** *
  -0 744311110000000000000000000000   30  +--0--+         |          * ***********+
  -1 520                               3     *        -15+   *   * *     ++++
     ----+----+----+----+----+----+                        +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**+01                                  -2        -1         0        +1        +2
```

Figure 2. SAS Univariate summary of distribution of percent difference for α (PCALPHA) from Elec-HO sample: RPRISM vs. PRISM

```
VARIABLE=PCBETA

                MOMENTS                                          QUANTILES(DEF=4)                           EXTREMES

N                  50  SUM WGTS            50  100% MAX     8.37182      99%     8.37182      LOWEST    HIGHEST
MEAN         -1.33395  SUM           -66.6977   75% Q3     0.816498      95%      6.3743    -27.7503    4.54824
STD DEV       6.04505  VARIANCE       36.5426   50% MED  -0.0537566      90%     4.51754    -21.1306     4.7648
SKEWNESS     -2.41091  KURTOSIS       8.54006   25% Q1     -2.06319      10%    -6.10101    -11.1111    5.20082
USS           1879.56  CSS            1790.59    0% MIN    -27.7503       5%    -15.6199    -9.76784    7.80856
CV           -453.168  STD MEAN      0.854899                             1%    -27.7503      -6.169    8.37182
T:MEAN=0     -1.56036  PROB>|T|       0.12511  RANGE        36.1221
SGN RANK         -118  PROB>|S|      0.213719  Q3-Q1        2.87969
NUM ¬= 0           47                          MODE               0

STEM LEAF                    #  BOXPLOT                    NORMAL PROBABILITY PLOT
   8 4                       1     0           9+                                             +++    *
   6 8                       1     0            |                                         +++   *
   4 02582                   5     0            |                                     +++++ **
   2 09                      2     |            |                                 +++ **
   0 12345688995            11  +-----+         |                            *********
  -0 866211198775100000     18  *--+--*         |                    *********++
  -2 38                      2  +-----+         |                   **   +++
  -4 53322                   5     |            |               **** +++
  -6 2                       1     |            |              * ++++
  -8 8                       1     0          -9+            *+++
 -10 1                       1     *            |          +*+
 -12                                            |       +++
 -14                                            | ++++
 -16                                            |+
 -18                                            |
 -20 1                       1     *            |          *
 -22                                            |
 -24                                            |
 -26 8                       1     *         -27+   *
     ----+----+----+----+                        +----+----+----+----+----+----+----+----+----+----+
                                                     -2        -1         0        +1        +2
```

Figure 3. SAS Univariate summary of distribution of percent difference for β (PCBETA) from Elec-HO sample: RPRISM vs. PRISM

```
VARIABLE=PCNAC

                 MOMENTS                                    QUANTILES(DEF=4)                          EXTREMES

N                 50  SUM WGTS          50  100% MAX    8.42912       99%      8.42912       LOWEST     HIGHEST
MEAN       -0.138382  SUM         -6.91911   75% Q3    0.330604       95%      2.12148     -5.52771     1.53022
STD DEV      1.82203  VARIANCE      3.3198   50% MED  -0.140715       90%      1.50557     -2.93059     1.65187
SKEWNESS     1.56312  KURTOSIS      10.377   25% Q1   -0.873389       10%      -1.9762      -2.8264      1.9099
USS          163.628  CSS           162.67    0% MIN   -5.52771        5%     -2.87329     -2.23295     2.38008
CV          -1316.67  STD MEAN    0.257674                             1%     -5.52771     -2.01526     8.42912
T:MEAN=0   -0.537043  PROB>|T|    0.593668  RANGE       13.9568
SGN RANK        -110  PROB>|S|     0.24656  Q3-Q1       1.20399
NUM ^= 0          47                        MODE              0

STEM LEAF                   #  BOXPLOT                       NORMAL PROBABILITY PLOT
   8 4                      1     *        8.5+                                                        *
   7                                           |
   6                                           |
   5                                           |
   4                                           |                                                        +++
   3                                           |                                                   +++++
   2 4                      1     0            |                                              ++++++*
   1 023579                 6     |        1.5+                                         +++++++ **
   0 1123334579            10  +-----+         |                                   +*********
  -0 986554333222111100000 21  *--+--*         |                           **********
  -1 665431                 6     |            |                     *****+++
  -2 9820                   4     0            |                * **+*++
  -3                                           |             ++++++
  -4                                           |++++
  -5 5                      1     *       -5.5+   *
     ----+----+----+----+-                      +----+----+----+----+----+----+----+----+----+----+
                                                     -2        -1         0        +1        +2
```

Figure 4. SAS Univariate summary of distribution of percent difference for NAC (PCNAC) from Elec-HO sample: RPRISM vs. PRISM

```
VARIABLE=PCBHO

              MOMENTS                                        QUANTILES(DEF=4)                    EXTREMES

N                50  SUM WGTS          50  100% MAX    70.463      99%      70.463     LOWEST    HIGHEST
MEAN        1.28554  SUM          64.2772   75% Q3   0.908801      95%     23.9696   -21.5682    3.87564
STD DEV     12.1592  VARIANCE     147.846   50% MED -0.0549775     90%      3.8332   -12.8196    4.78354
SKEWNESS    4.21224  KURTOSIS     22.7859   25% Q1   -1.64653      10%    -3.50894   -8.07451    14.6172
USS         7327.08  CSS          7244.44    0% MIN  -21.5682       5%    -10.2098   -7.70118    35.4004
CV           945.84  STD MEAN     1.71957                           1%    -21.5681   -3.52718     70.463
T:MEAN=0   0.747597  PROB>|T|    0.458276  RANGE      92.0311
SGN RANK        -68  PROB>|S|    0.475044  Q3-Q1      2.55533
NUM ¬= 0         47                        MODE             0

STEM LEAF                           #  BOXPLOT                          NORMAL PROBABILITY PLOT
   7 0                              1     *         72.5+                                                    *
   6                                                    |
   6                                                    |
   5                                                    |
   5                                                    |
   4                                                    |
   4                                                    |
   3 5                              1     *             |                                                *
   3                                                    |                                                    ++
   2                                                    |                                                ++++
   2                                                    |                                            ++++
   1 5                              1                   |                                        ++++
   1                                      *             |                                    ++++    *
   0 5                              1                   |                                ++++
   0 1111111112234                 13  +--0--+          |                        ++++++++++++++++ *
  -0 43333222211111111000000000000 29  *-----*          |              **************
  -0 88                             2     0             |           **     ++++
  -1 3                              1     *             |          *   ++++
  -1                                                    |            ++++
  -2 2                              1     *        -22.5+      *++++
     ----+----+----+----+----+----                      +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**+01                                   -2        -1         0        +1        +2
```

Figure 5. SAS Univariate summary of distribution of percent difference for βH_o (PCBHO) from Elec-HO sample: RPRISM vs. PRISM

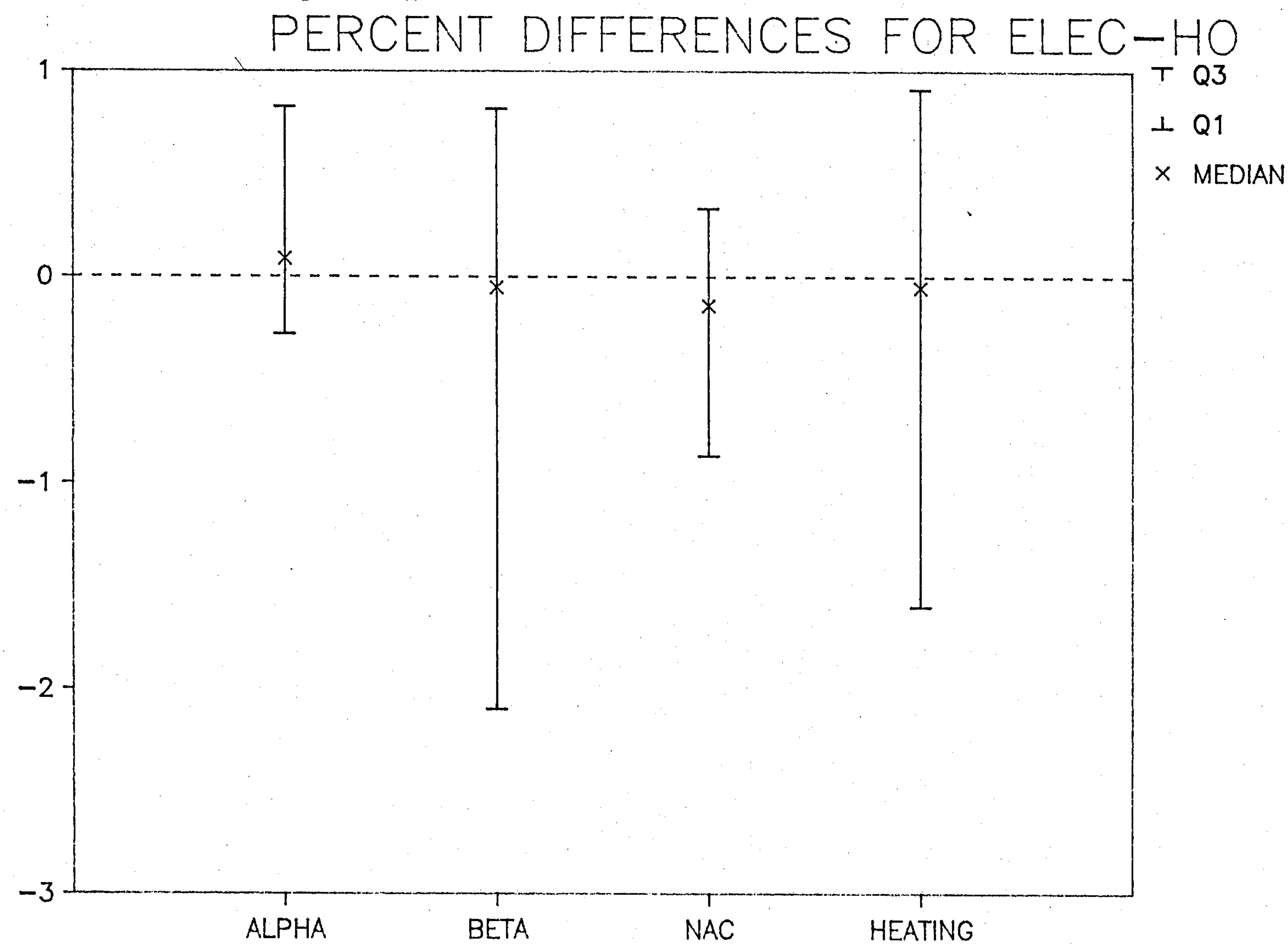


Figure 6. Plot of medians and quartiles of percent differences of PRISM vs. RPRISM parameters for Elec-HO sample.

```
VARIABLE=CVRALPHA

              MOMENTS                                        QUANTILES(DEF=4)                     EXTREMES

N                  50  SUM WGTS          50 100% MAX   24.7267      99%     24.7266      LOWEST     HIGHEST
MEAN          1.73896  SUM          86.9481  75% Q3    1.40775      95%     3.40472    0.878932     1.90952
STD DEV       3.35989  VARIANCE     11.2888  50% MED    1.0789      90%     1.90546    0.911385     2.30541
SKEWNESS       6.8061  KURTOSIS     47.3578  25% Q1   0.968316      10%    0.926783     0.91634     3.38156
USS           704.353  CSS          553.153   0% MIN  0.878932       5%     0.91411    0.921042     3.43303
CV            193.212  STD MEAN     0.47516                          1%    0.878932    0.926743     24.7267
T:MEAN=0      3.65974  PROB>|T| .000618197  RANGE      23.8477
SGN RANK        637.5  PROB>|S|      0.0001  Q3-Q1     0.439439
NUM ¬= 0           50                        MODE             1

STEM LEAF                                       #  BOXPLOT                 NORMAL PROBABILITY PLOT
  24 7                                          1     *         25+                                                *
  22                                                              |
  20                                                              |
  18                                                              |
  16                                                              |
  14                                                              |
  12                                                            13+
  10                                                              |                                                  +
   8                                                              |                                            ++++++
   6                                                              |                                      ++++++
   4                                                              |                                ++++++
   2 344                                        3     0           |                          ++++++          ** *
   0 9999999990000000000000001111112223334444566899    46  *--+--*          1+   *    * **  **************************
     ----+----+----+----+----+----+----+----+----+-                         +----+----+----+----+----+----+----+----+----+----+
                                                                                -2        -1         0        +1        +2
```

Figure 7. SAS Univariate summary of distribution of CVRs for α (CVRALPHA) from Elec-HO sample.

```
VARIABLE=CVRBETA

                 MOMENTS                                        QUANTILES(DEF=4)                          EXTREMES

N                 50  SUM WGTS           50 100% MAX   6.50247       99%      6.50247      LOWEST     HIGHEST
MEAN         1.36428  SUM            68.214  75% Q3    1.45981       95%      2.68467    0.865575       2.034
STD DEV     0.857859  VARIANCE     0.735922  50% MED   1.10693       90%      2.00856     0.87633     2.40358
SKEWNESS     4.73116  KURTOSIS        26.87  25% Q1     0.9839       10%     0.926715    0.910883     2.56071
USS          129.123  CSS           36.0602   0% MIN  0.865575        5%     0.895334    0.921541     2.83618
CV           62.8799  STD MEAN      0.12132                           1%     0.865575    0.926354     6.50247
T:MEAN=0     11.2453  PROB>|T|       0.0001  RANGE      5.6369
SGN RANK       637.5  PROB>|S|       0.0001  Q3-Q1     0.47591
NUM ¬= 0          50                         MODE            1

STEM LEAF                          #  BOXPLOT                           NORMAL PROBABILITY PLOT
   6 5                             1     *       6.75+                                                      *
   6                                                 |
   5                                                 |
   5                                                 |
   4                                                 |
   4                                                 |
   3                                             3.75+                                                          +
   3                                                 |                                                     +++++
   2 68                            2     0           |                                               ++*+*+
   2 04                            2     0           |                                         +++++* *
   1 55555678                      8     |           |                                   ++++++ ****
   1 0000000000000001111222333344 28  *--+--*        |                       ****************
   0 999999999                     9  +-----+    0.75+   *    * ** *********+
     ----+----+----+----+----+---                     +----+----+----+----+----+----+----+----+----+----+
                                                          -2        -1         0        +1        +2
```

Figure 8. SAS Univariate summary of distribution of CVRs for β (CVRBETA) from Elec-HO sample.

```
VARIABLE=CVRNAC

                    MOMENTS                                      QUANTILES(DEF=4)                          EXTREMES

N                 50  SUM WGTS            50  100% MAX   4.62454      99%    4.62454      LOWEST     HIGHEST
MEAN         1.31258  SUM            65.6291   75% Q3    1.39519      95%    2.81913    0.879171     1.81185
STD DEV     0.640783  VARIANCE      0.410602   50% MED   1.11241      90%    1.80221    0.879884     2.09349
SKEWNESS     3.54037  KURTOSIS       15.1362   25% Q1   0.978022      10%   0.923211    0.917895      2.5933
USS          106.263  CSS            20.1195    0% MIN  0.879171       5%    0.90079    0.920747     3.09515
CV           48.8185  STD MEAN     0.0906203                           1%   0.879171    0.922762     4.62454
T:MEAN=0     14.4844  PROB>|T|        0.0001  RANGE      3.74537
SGN RANK       637.5  PROB>|S|        0.0001  Q3-Q1     0.417172
NUM ¬= 0          50                          MODE      0.879171

STEM LEAF                     #  BOXPLOT                          NORMAL PROBABILITY PLOT
  46 2                        1     *         4.7+                                                    *
  44                                             |
  42                                             |
  40                                             |
  38                                             |
  36                                             |
  34                                             |
  32                                             |
  30 0                        1     *            |                                                *
  28                                             |                                                    ++
  26                                             |                                                 +++
  24 9                        1     0            |                                              *++++
  22                                             |                                            +++
  20 9                        1     0            |                                         +++ *
  18 1                        1     |            |                                      +++  *
  16 072                      3     |            |                                   +++  ***
  14 0342                     4     |            |                                +++   **
  12 4788579                  7  +--+--+         |                             +++ *****
  10 00000245781245799       17  *-----*         |                          ********
   8 88222334466889          14  +-----+      0.9+   *   * ** **********
     ----+----+----+----+                         +----+----+----+----+----+----+----+----+----+----+
   MULTIPLY STEM.LEAF BY 10**-01                       -2        -1         0        +1        +2
```

Figure 9. SAS Univariate summary of distribution of CVRs for NAC (CVRNAC) from Elec-HO sample.

```
VARIABLE=CVRBHO

                 MOMENTS                                      QUANTILES(DEF=4)                         EXTREMES

N                50  SUM WGTS           50  100% MAX   7.56411      99%    7.56411      LOWEST    HIGHEST
MEAN        1.44749  SUM           72.3746   75% Q3    1.32531      95%    4.08634    0.879089    2.24114
STD DEV     1.13296  VARIANCE      1.28359   50% MED   1.12908      90%    2.20057    0.918631    2.81995
SKEWNESS    4.13786  KURTOSIS      19.0544   25% Q1   0.984768      10%   0.927465    0.923393    3.27926
USS         167.658  CSS           62.8959    0% MIN  0.879089       5%    0.92125    0.925549     5.0728
CV          78.2703  STD MEAN     0.160224                           1%   0.879089     0.92736    7.56411
T:MEAN=0    9.03417  PROB>|T|       0.0001  RANGE      6.68502
SGN RANK      637.5  PROB>|S|       0.0001  Q3-Q1      0.34054
NUM ¬= 0         50                         MODE      0.879089

STEM LEAF                                    #  BOXPLOT                        NORMAL PROBABILITY PLOT
   7 6                                       1     *         7.75+                                                    *
   7                                                             |
   6                                                             |
   6                                                             |
   5                                                             |
   5 1                                       1     *             |                                              *
   4                                                             |
   4                                                         4.25+                                                   +++
   3                                                             |                                               ++++
   3 3                                       1     *             |                                          ++*++
   2 8                                       1     *             |                                      ++++ *
   2 2                                       1     0             |                                 +++++    *
   1 5688                                    4     |             |                             ++++     ***
   1 000000000000011111122222222333444      33  *--+--*          |                        **************
   0 99999999                                8  +-----+      0.75+   *    * ** *********+
     ----+----+----+----+----+----+---                            +----+----+----+----+----+----+----+----+----+----+
                                                                      -2        -1         0        +1        +2
```

Figure 10. SAS Univariate summary of distribution of CVRs for βH_o (CVRBHO) from Elec-HO sample.

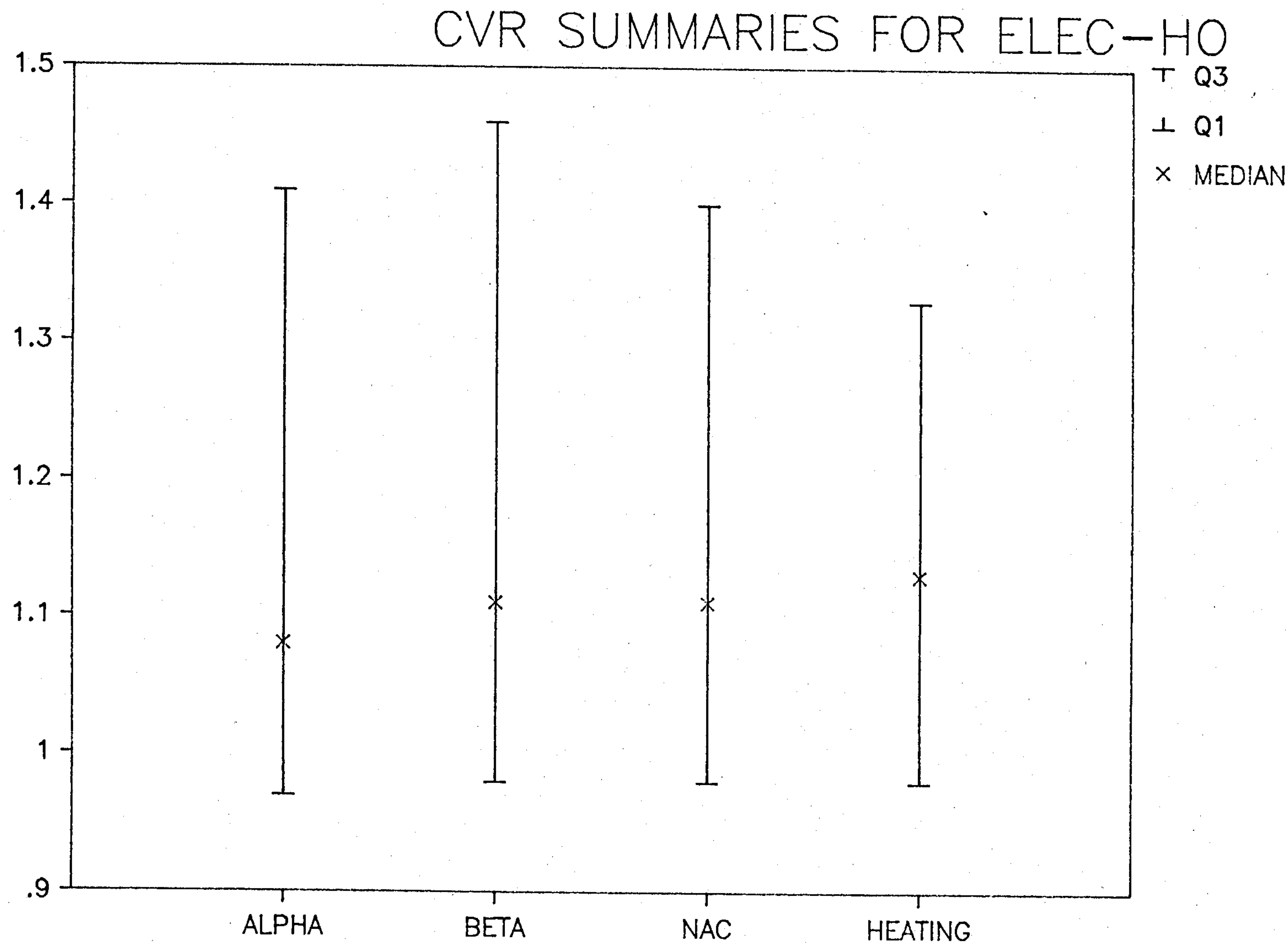


Figure 11. Plot of medians and quartiles of CVRs of PRISM vs. RPRISM parameters for Elec-HO sample.

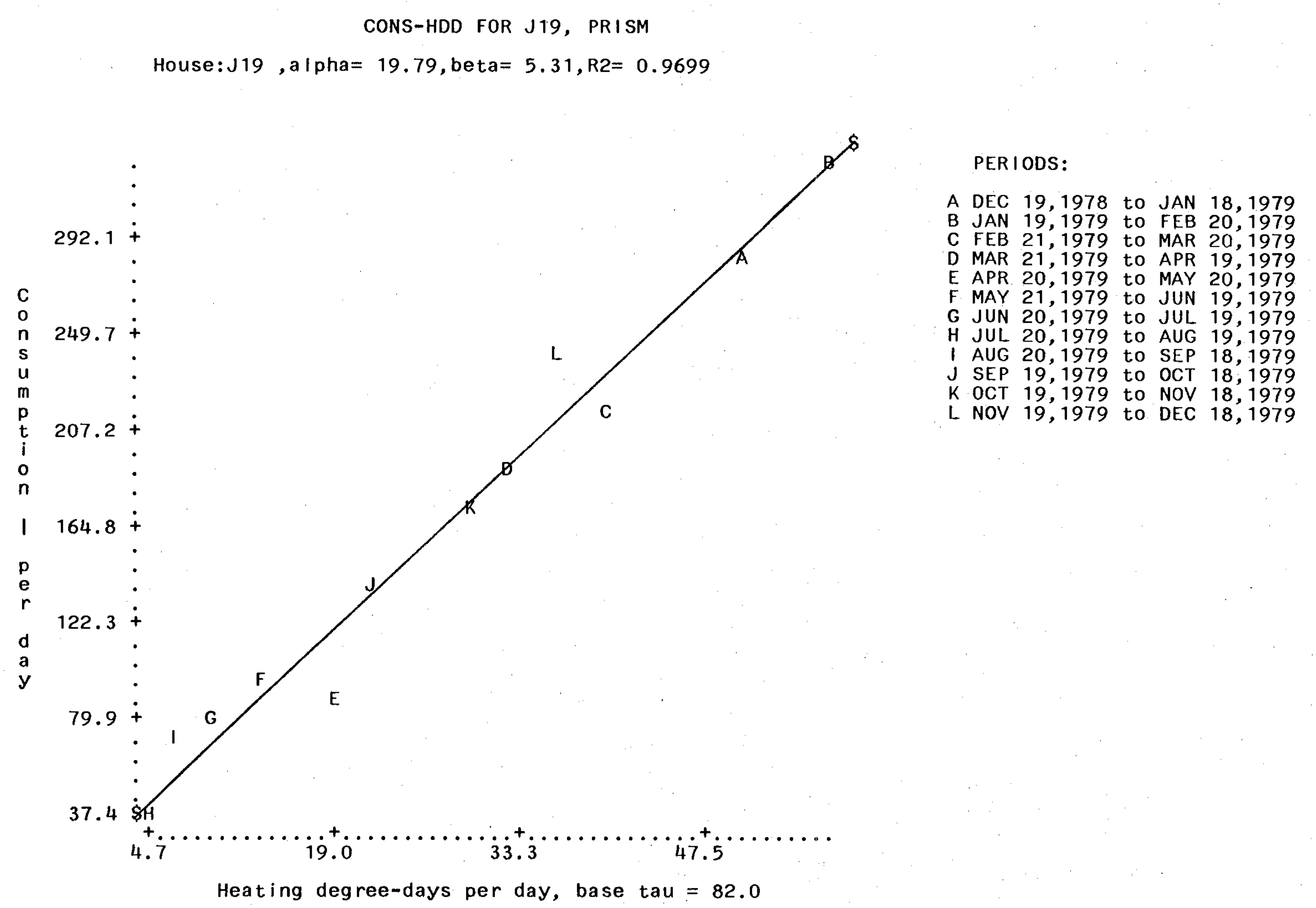


Figure 12. PRISM plot of consumption vs. heating-degree days for House J19 from Elec-HO sample.

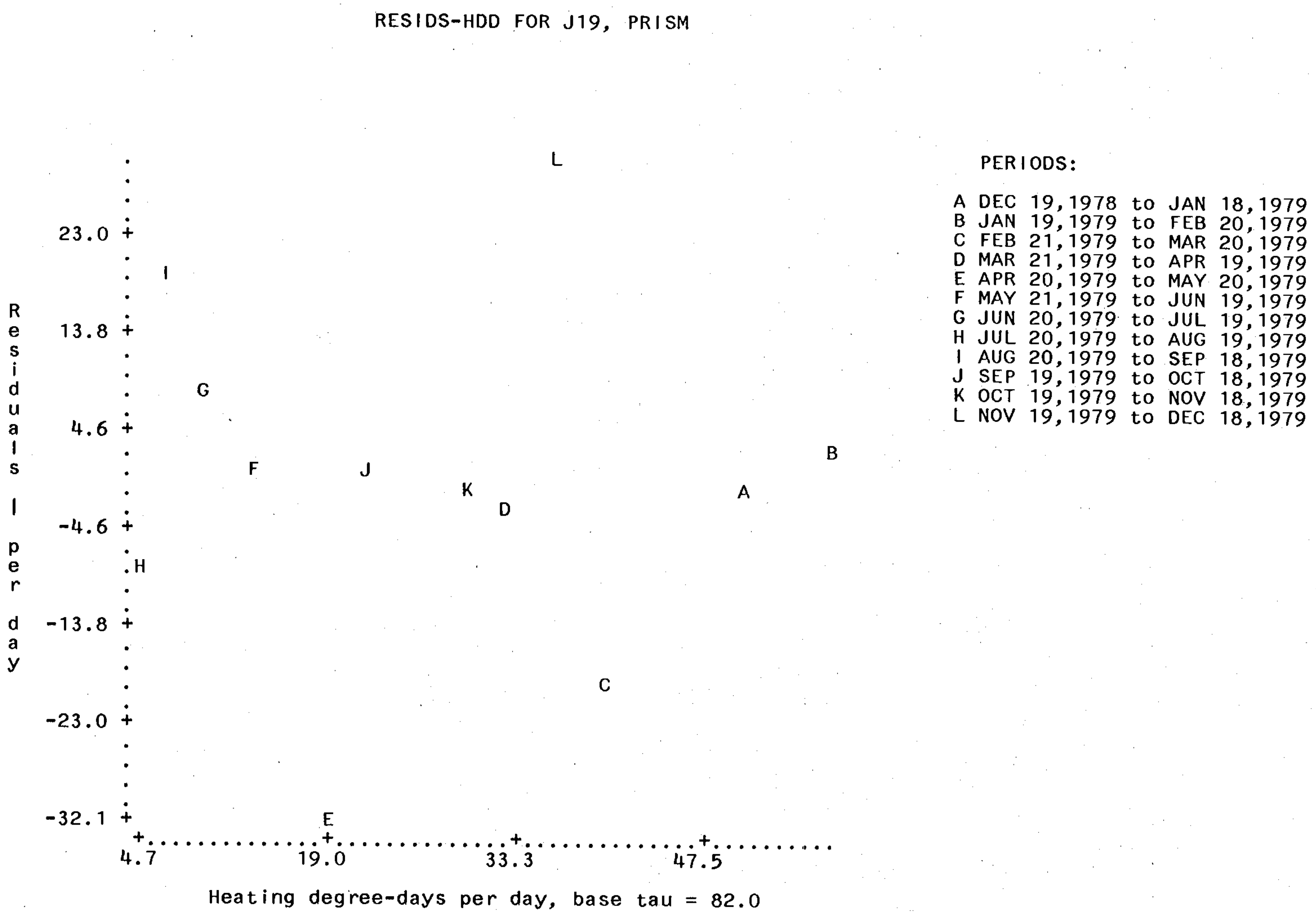
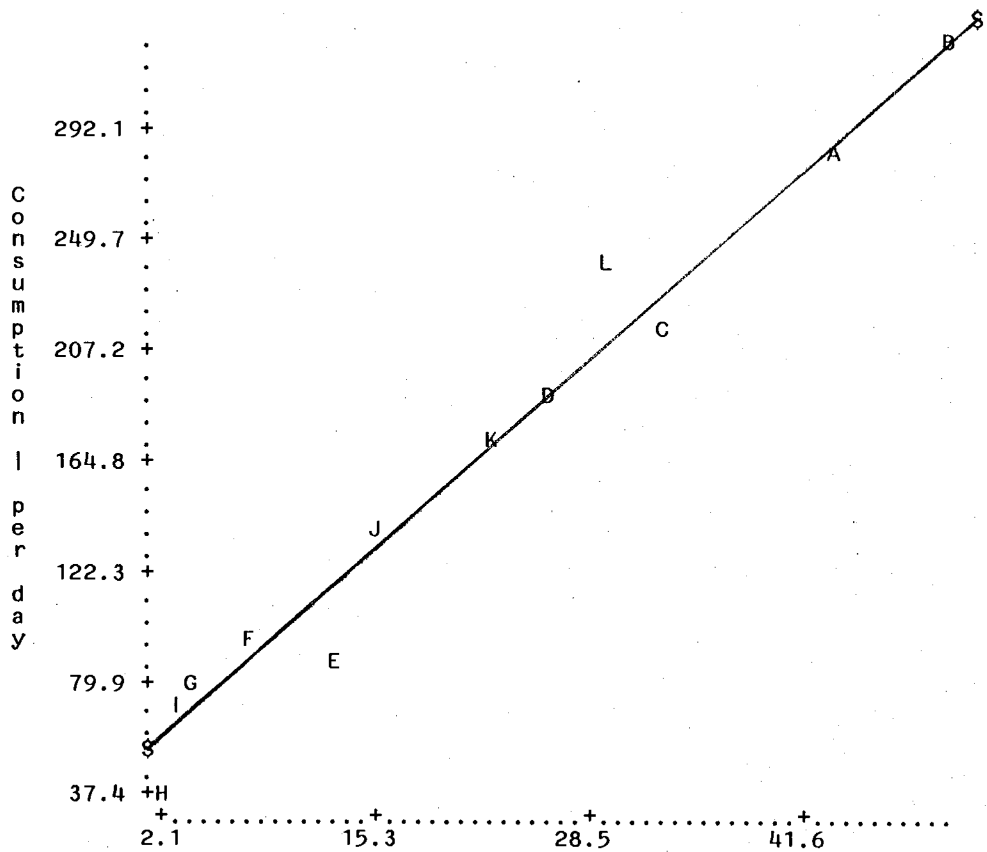


Figure 13. PRISM plot of residuals vs. heating degree-days for House J19 from Elec-HO sample.

CONS-HDD FOR J19, RPRISM

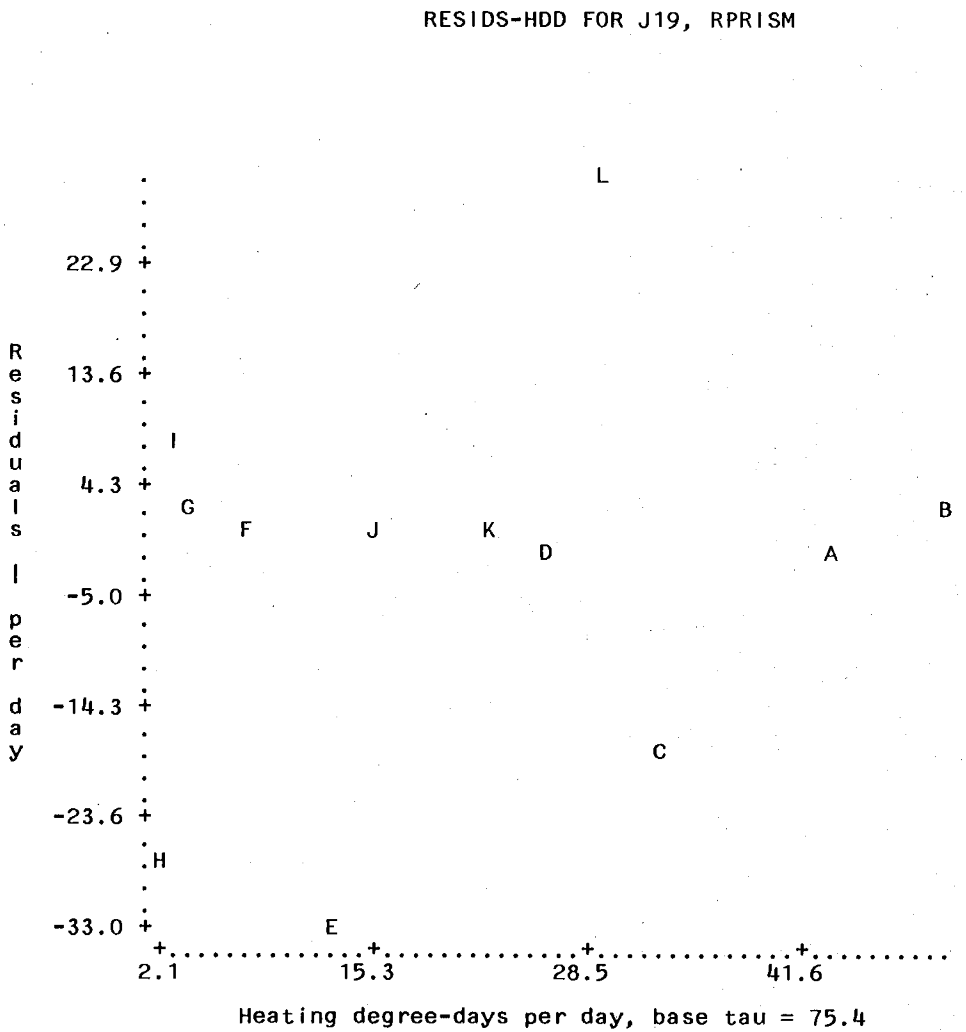
House:J19 ,alpha= 53.68,beta= 5.35,R2= 0.9920



PERIODS:

A DEC 19,1978 to JAN 18,1979
B JAN 19,1979 to FEB 20,1979
C FEB 21,1979 to MAR 20,1979
D MAR 21,1979 to APR 19,1979
E APR 20,1979 to MAY 20,1979
F MAY 21,1979 to JUN 19,1979
G JUN 20,1979 to JUL 19,1979
H JUL 20,1979 to AUG 19,1979
I AUG 20,1979 to SEP 18,1979
J SEP 19,1979 to OCT 18,1979
K OCT 19,1979 to NOV 18,1979
L NOV 19,1979 to DEC 18,1979

Figure 14. RPRISM plot of consumption vs. heating degree-days for House J19 from Elec-HO sample.



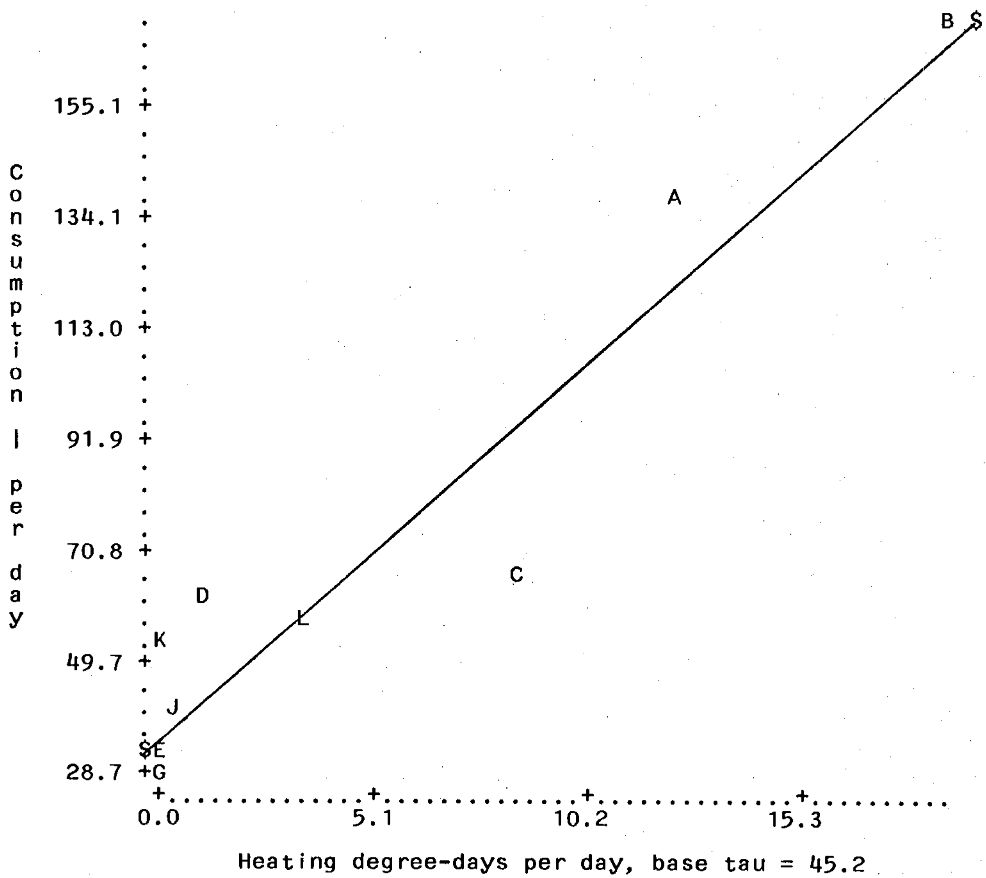
PERIODS:

A DEC 19,1978 to JAN 18,1979
B JAN 19,1979 to FEB 20,1979
C FEB 21,1979 to MAR 20,1979
D MAR 21,1979 to APR 19,1979
E APR 20,1979 to MAY 20,1979
F MAY 21,1979 to JUN 19,1979
G JUN 20,1979 to JUL 19,1979
H JUL 20,1979 to AUG 19,1979
I AUG 20,1979 to SEP 18,1979
J SEP 19,1979 to OCT 18,1979
K OCT 19,1979 to NOV 18,1979
L NOV 19,1979 to DEC 18,1979

Figure 15. RPRISM plot of residuals vs. heating degree-days for House J19 from Elec-HO sample.

CONS-HDD FOR J43, PRISM

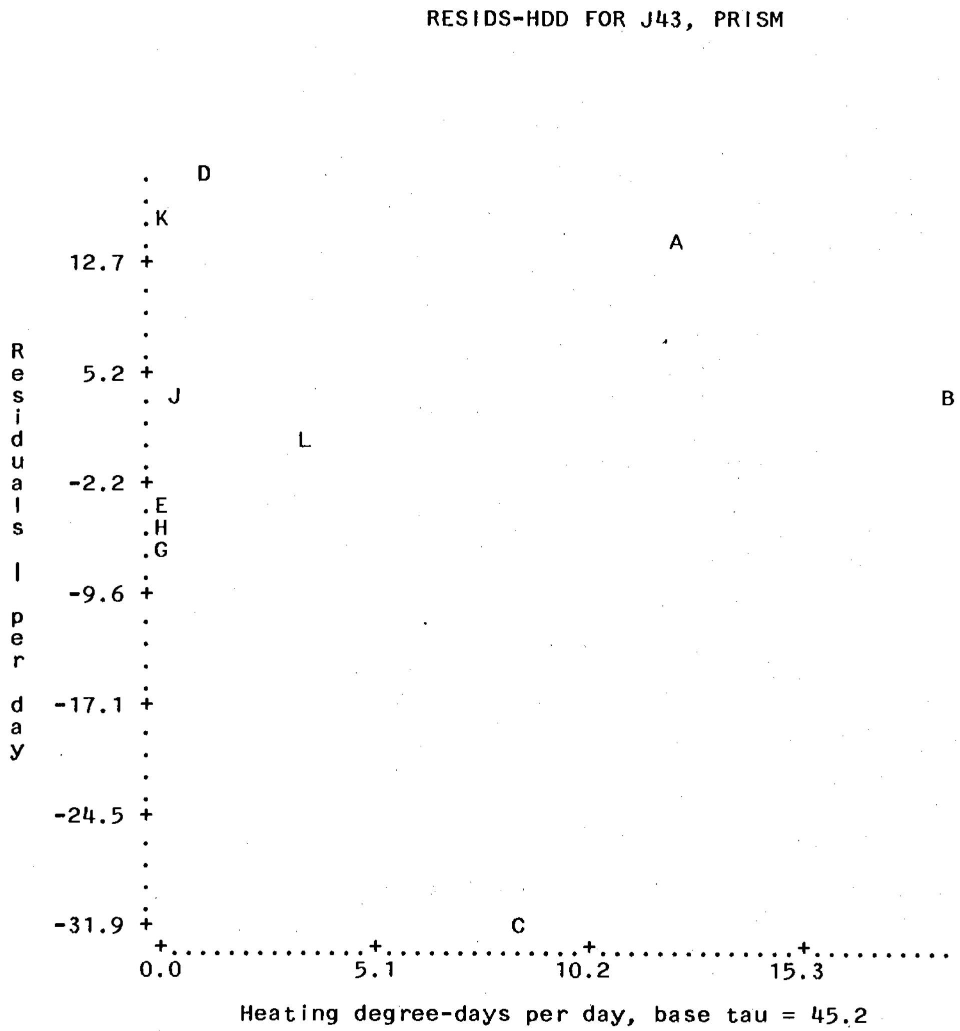
House:J43 ,alpha= 35.94,beta= 7.08,R2= 0.9153



PERIODS:

A DEC 15,1978 to JAN 16,1979
B JAN 17,1979 to FEB 15,1979
C FEB 16,1979 to MAR 18,1979
D MAR 19,1979 to APR 17,1979
E APR 18,1979 to MAY 16,1979
E MAY 17,1979 to JUN 17,1979
G JUN 18,1979 to JUL 17,1979
G JUL 18,1979 to AUG 15,1979
E AUG 16,1979 to SEP 16,1979
J SEP 17,1979 to OCT 16,1979
K OCT 17,1979 to NOV 14,1979
L NOV 15,1979 to DEC 16,1979

Figure 16. PRISM plot of consumption vs. heating-degree days for House J43 from Elec-HO sample.



PERIODS:

A DEC 15,1978 to JAN 16,1979
B JAN 17,1979 to FEB 15,1979
C FEB 16,1979 to MAR 18,1979
D MAR 19,1979 to APR 17,1979
E APR 18,1979 to MAY 16,1979
E MAY 17,1979 to JUN 17,1979
G JUN 18,1979 to JUL 17,1979
H JUL 18,1979 to AUG 15,1979
E AUG 16,1979 to SEP 16,1979
J SEP 17,1979 to OCT 16,1979
K OCT 17,1979 to NOV 14,1979
L NOV 15,1979 to DEC 16,1979

Figure 17. PRISM plot of residuals vs. heating degree-days for House J43 from Elec-HO sample.

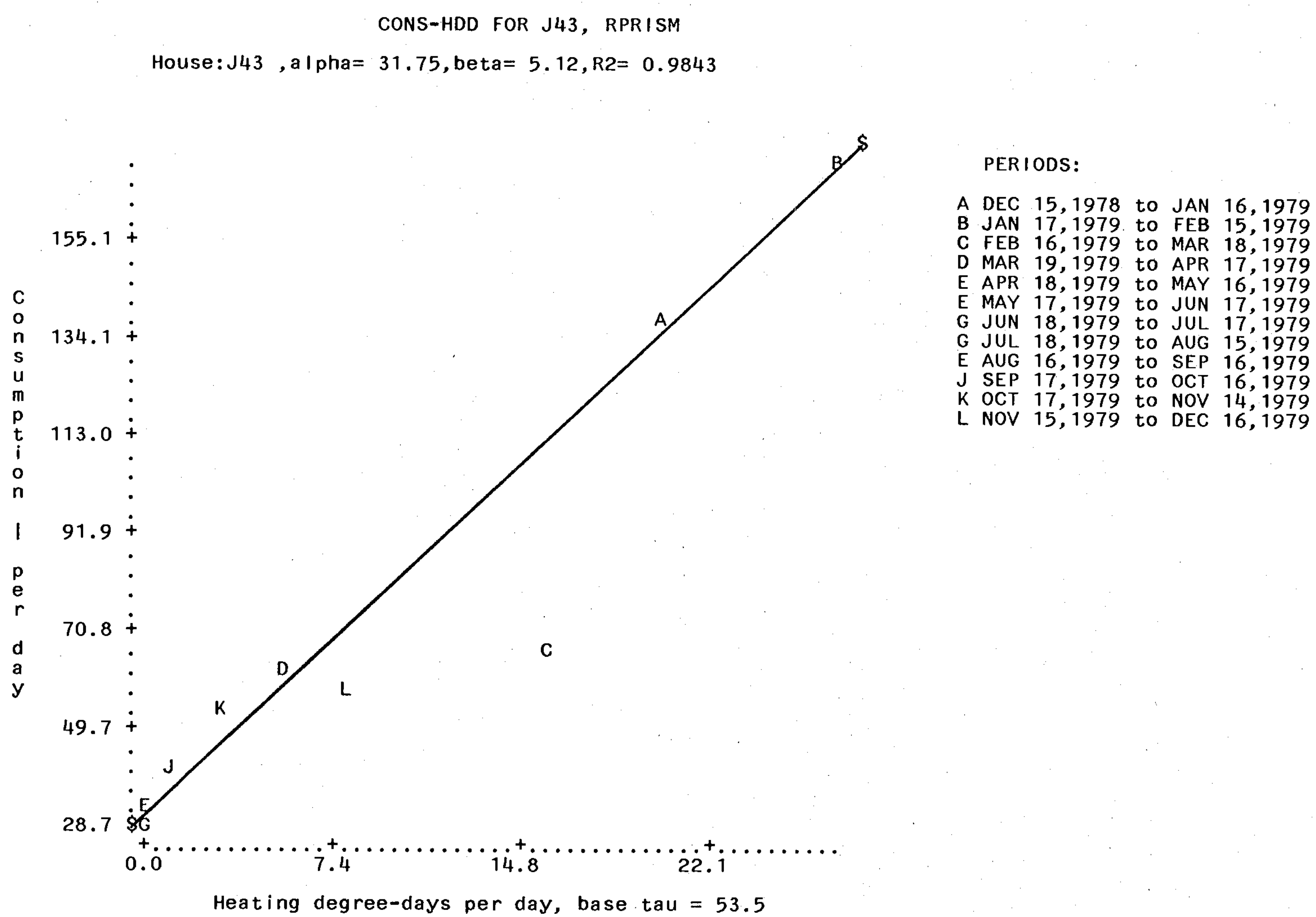
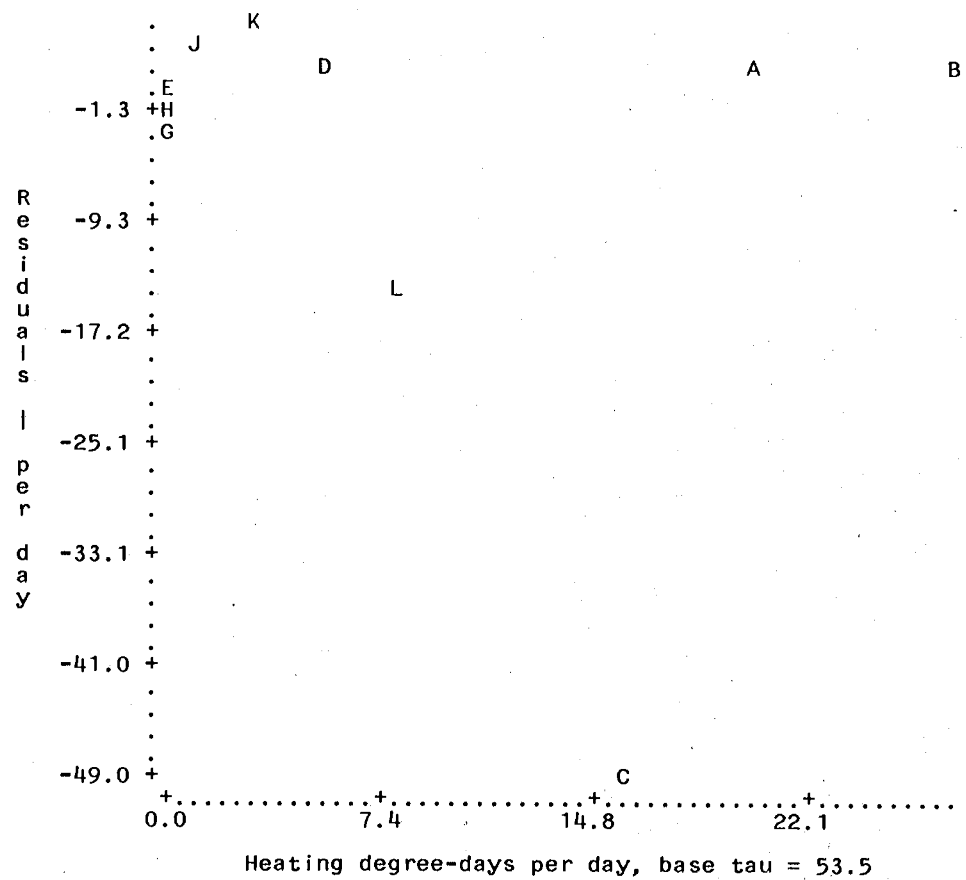


Figure 18. RPRISM plot of consumption vs. heating degree-days for House J43 from Elec-HO sample.

RESIDS-HDD FOR J43, RPRISM



PERIODS:

A DEC 15,1978 to JAN 16,1979
B JAN 17,1979 to FEB 15,1979
C FEB 16,1979 to MAR 18,1979
D MAR 19,1979 to APR 17,1979
E APR 18,1979 to MAY 16,1979
E MAY 17,1979 to JUN 17,1979
G JUN 18,1979 to JUL 17,1979
H JUL 18,1979 to AUG 15,1979
E AUG 16,1979 to SEP 16,1979
J SEP 17,1979 to OCT 16,1979
K OCT 17,1979 to NOV 14,1979
L NOV 15,1979 to DEC 16,1979

Figure 19. RPRISM plot of residuals vs. heating degree-days for House J43 from Elec-HO sample.

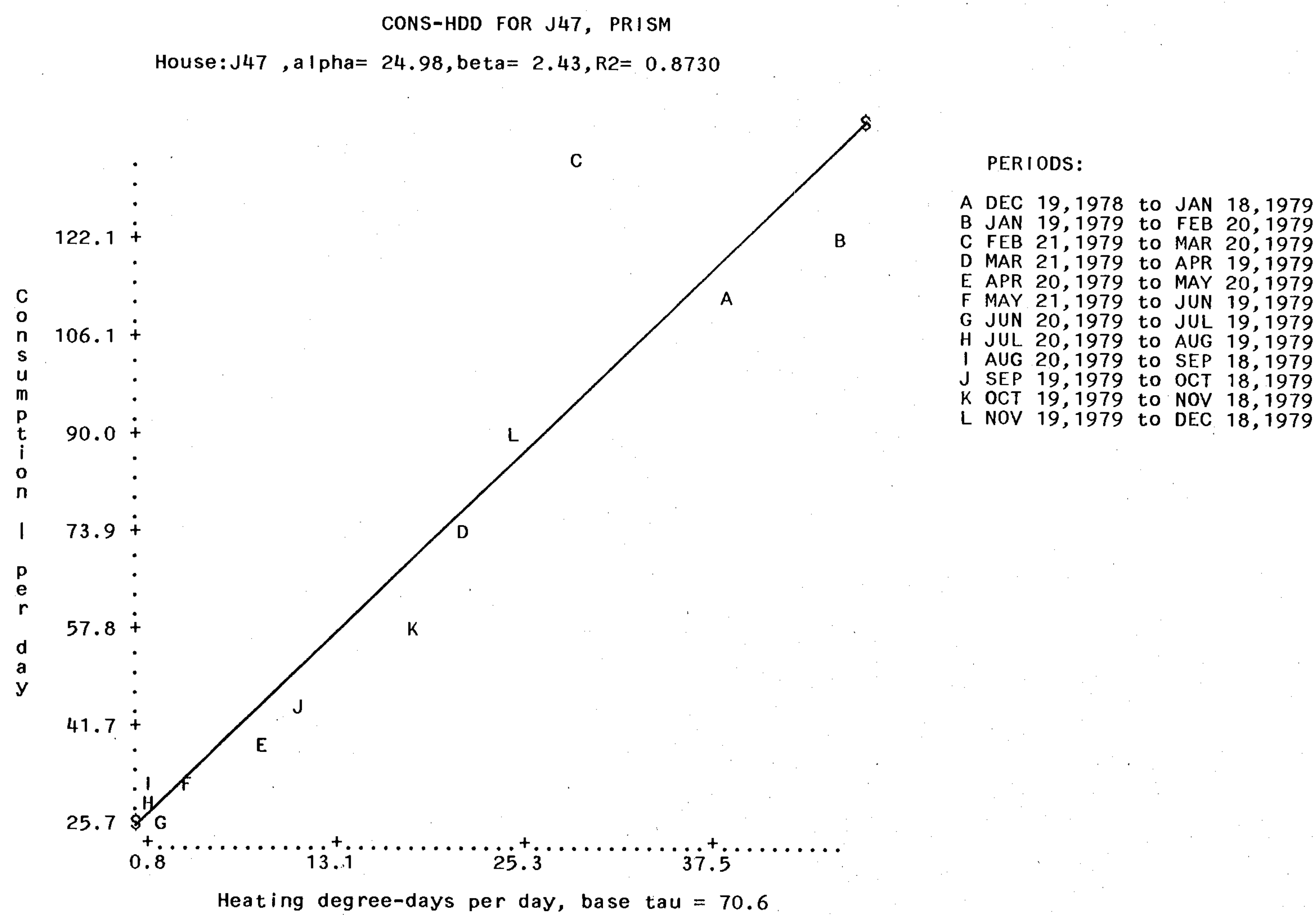
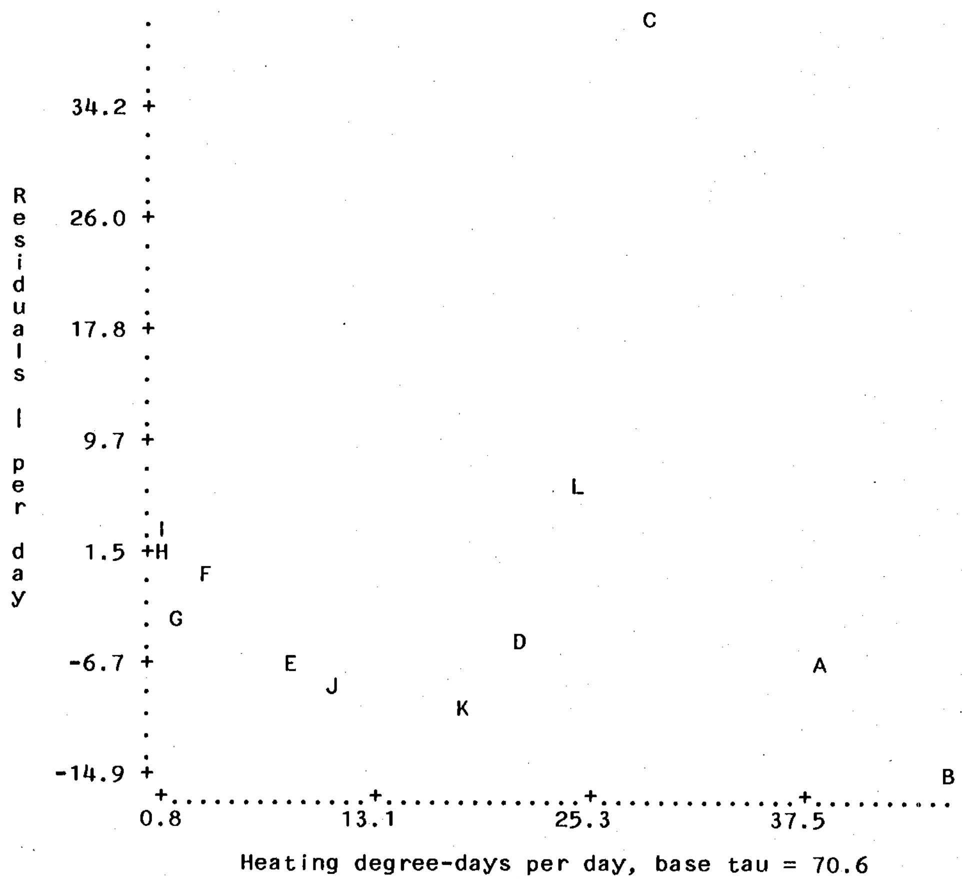


Figure 20. PRISM plot of consumption vs. heating-degree days for House J47 from Elec-HO sample.

RESIDS-HDD FOR J47, PRISM



PERIODS:

A DEC 19,1978 to JAN 18,1979
B JAN 19,1979 to FEB 20,1979
C FEB 21,1979 to MAR 20,1979
D MAR 21,1979 to APR 19,1979
E APR 20,1979 to MAY 20,1979
F MAY 21,1979 to JUN 19,1979
G JUN 20,1979 to JUL 19,1979
H JUL 20,1979 to AUG 19,1979
I AUG 20,1979 to SEP 18,1979
J SEP 19,1979 to OCT 18,1979
K OCT 19,1979 to NOV 18,1979
L NOV 19,1979 to DEC 18,1979

Figure 21. PRISM plot of residuals vs. heating degree-days for House J47 from Elec-HO sample.

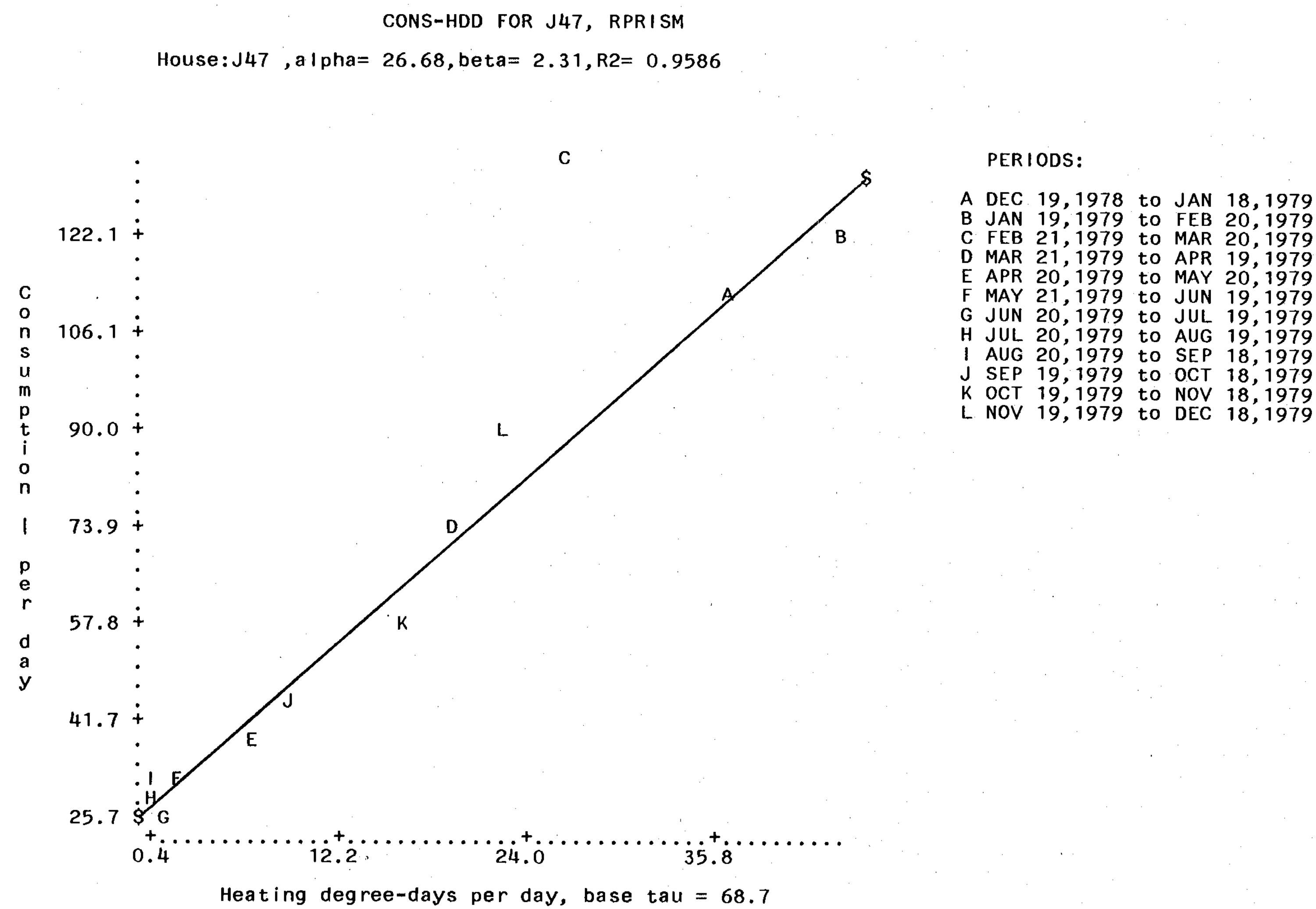
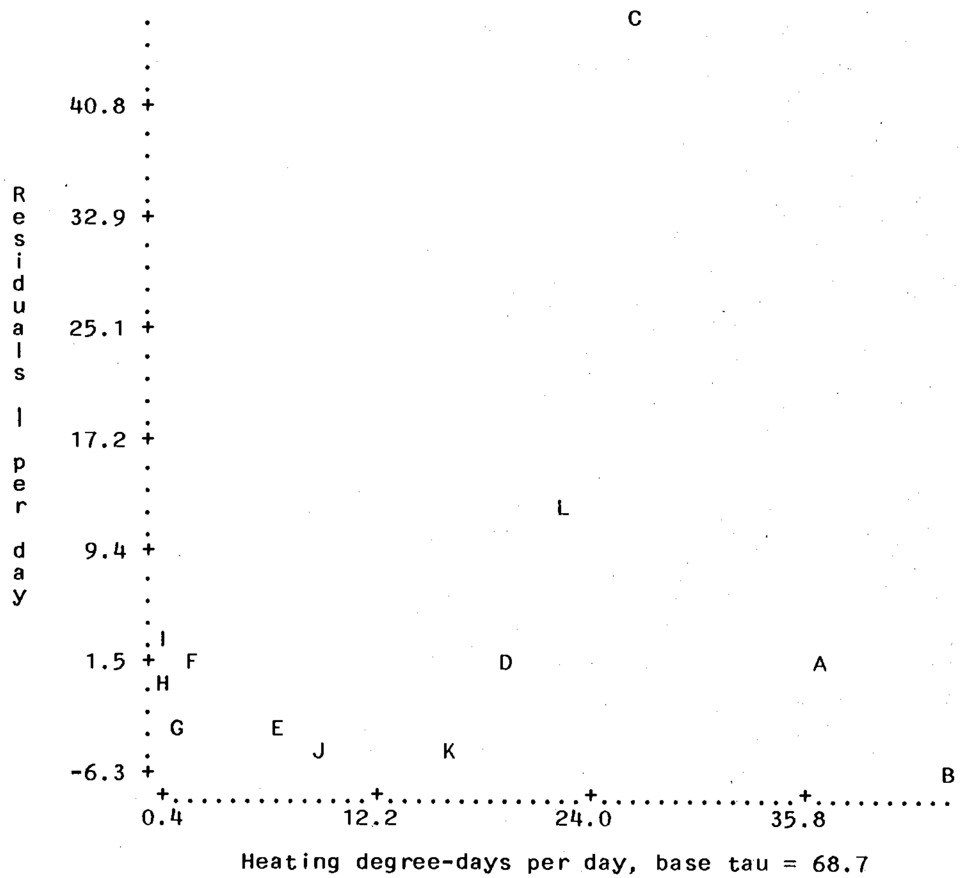


Figure 22. RPRISM plot of consumption vs. heating degree-days for House J47 from Elec-HO sample.

FIGURE 28: RESIDS-HDD FOR J47, RPRISM



PERIODS:

A DEC 19,1978 to JAN 18,1979
B JAN 19,1979 to FEB 20,1979
C FEB 21,1979 to MAR 20,1979
D MAR 21,1979 to APR 19,1979
E APR 20,1979 to MAY 20,1979
F MAY 21,1979 to JUN 19,1979
G JUN 20,1979 to JUL 19,1979
H JUL 20,1979 to AUG 19,1979
I AUG 20,1979 to SEP 18,1979
J SEP 19,1979 to OCT 18,1979
K OCT 19,1979 to NOV 18,1979
L NOV 19,1979 to DEC 18,1979

Figure 23. RPRISM plot of residuals vs. heating degree-days for House J47 from Elec-HO sample.

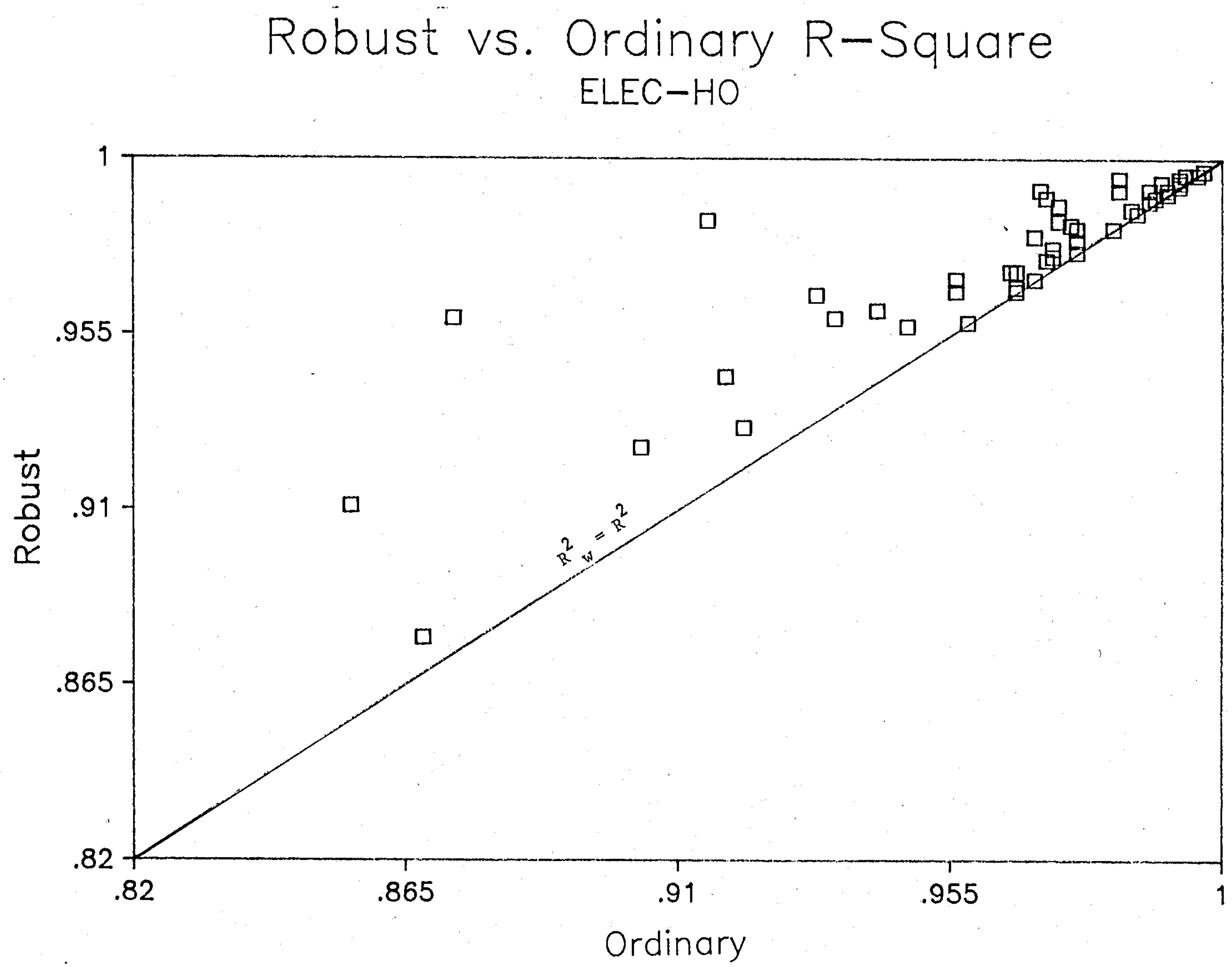


Figure 24. R^2 estimates for Robust (RPRISM) vs. Ordinary PRISM for Elec-HO sample.

```
VARIABLE=PCALPHA

                MOMENTS                                     QUANTILES(DEF=4)                      EXTREMES

N                 50  SUM WGTS          50  100% MAX    2.03571     99%     2.03571     LOWEST    HIGHEST
MEAN        -2.30721  SUM         -115.361   75% Q3    0.724495     95%     1.53939   -12.9725    1.02985
STD DEV      3.93168  VARIANCE     15.4581   50% MED  -0.573236     90%     1.02624   -12.4166    1.08165
SKEWNESS     -1.3744  KURTOSIS     1.02035   25% Q1    -3.94541     10%    -9.98623   -10.8894    1.50194
USS          1023.61  CSS          757.449    0% MIN   -12.9725      5%    -11.5767   -10.7162    1.58516
CV          -170.408  STD MEAN    0.556024                          1%    -12.9725   -9.98775    2.03571
T:MEAN=0    -4.14949  PROB>|T|  .000132208  RANGE       15.0082
SGN RANK      -338.5  PROB>|S|  .000773281  Q3-Q1        4.6699
NUM ¬= 0          49                        MODE       -12.9725

STEM LEAF                     #  BOXPLOT                        NORMAL PROBABILITY PLOT
   2 0                        1     |          2.5+                                         +++       *
   1 000156                   6     |             |                                      ++  * ** *
   0 24788889                 8  +-----+          |                                  ********
  -0 66543221100             11  *-----*      -0.5+                             *****+
  -1 887433                   6  |     |          |                          *** +++
  -2 876                      3  |  +  |          |                        **  ++
  -3 832                      3  +-----+      -3.5+                     ** +++
  -4 54                       2     |             |                     ****
  -5 963                      3     |             |                   ****
  -6 6                        1     |         -6.5+                  *+
  -7                                |             |               +++
  -8                                |             |            +++
  -9                                |         -9.5+         ++    **
 -10 9700                     4     |             |     +++    **
 -11                                              | ++
 -12 04                       2     0        -12.5++  *   *
     ----+----+----+----+                          +----+----+----+----+----+----+----+----+----+----+
                                                        -2        -1         0        +1        +2
```

Figure 25. SAS Univariate summary of distribution of percent difference for α (PCALPHA) from Elec-AC sample: RPRISM vs. PRISM

```
VARIABLE=PCBETA

                MOMENTS                                        QUANTILES(DEF=4)                    EXTREMES

N                 50  SUM WGTS          50 100% MAX   23.6502     99%     23.6502     LOWEST    HIGHEST
MEAN        0.145799  SUM          7.28996  75% Q3   0.787441     95%     14.1689   -22.3567    5.43393
STD DEV      6.34171  VARIANCE     40.2173  50% MED -0.114393     90%     5.40169   -10.1024     8.8463
SKEWNESS    0.620983  KURTOSIS     6.82204  25% Q1   -1.87962     10%    -4.14539   -7.80129    11.0686
USS          1971.71  CSS          1970.65   0% MIN  -22.3567      5%    -8.83679   -6.41204    17.9582
CV           4349.62  STD MEAN    0.896853                         1%    -22.3567   -4.17104    23.6502
T:MEAN=0    0.162568  PROB>|T|    0.871528  RANGE     46.0069
SGN RANK       -76.5  PROB>|S|    0.449653  Q3-Q1     2.66706
NUM ¬= 0          49                        MODE     -22.3567

STEM LEAF                          #  BOXPLOT                          NORMAL PROBABILITY PLOT
   2 4                             1     *          22.5+                                                  *
   1 8                             1     *              |                                              *     ++
   1 1                             1     *              |                                            *++++++++
   0 5559                          4     0           7.5+                                    +++****+*
   0 111111223                     9  +--+--+           |                            +***********
  -0 443333322111111111000000000000 30 *-----*           |               ***************
  -0 86                            2     0          -7.5+         **+++++++
  -1 0                             1     *              |  ++++++*++
  -1                                                    |++
  -2 2                             1     *         -22.5+   *
     ----+----+----+----+----+----+                     +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**+01                               -2        -1         0        +1        +2
```

Figure 26. SAS Univariate summary of distribution of percent difference for β (PCBETA) from Elec-AC sample: RPRISM vs. PRISM

```
VARIABLE=PCNAC

                MOMENTS                                           QUANTILES(DEF=4)                          EXTREMES

N                  50  SUM WGTS            50  100% MAX   4.05185      99%      4.05185       LOWEST     HIGHEST
MEAN        -0.512275  SUM           -25.6138   75% Q3   0.438869      95%      1.44689     -6.68634    0.958104
STD DEV       1.62296  VARIANCE       2.63401   50% MED -0.462711      90%       0.9536     -4.26543     0.98488
SKEWNESS     -1.04635  KURTOSIS       4.49915   25% Q1   -1.11233      10%     -2.37263     -3.73355     1.04106
USS           142.188  CSS            129.066    0% MIN  -6.68634       5%      -3.9729     -2.75795     1.94292
CV           -316.815  STD MEAN      0.229522                           1%     -6.68634     -2.42425     4.05185
T:MEAN=0     -2.23193  PROB>|T|     0.0302247  RANGE      10.7382
SGN RANK       -232.5  PROB>|S|     0.0210118  Q3-Q1       1.5512
NUM ¬= 0           49                          MODE      -6.68634

STEM LEAF                    #  BOXPLOT                         NORMAL PROBABILITY PLOT
   4 1                       1     0          4.25+                                                        *
   3                                              |                                                          +
   3                                              |                                                        +++
   2                                              |                                                     +++
   2                                              |                                                  +++
   1 9                       1     |              |                                               +++ *
   1 000                     3     |              |                                            +++  *
   0 5677899                 7     |              |                                       +******* *
   0 113344                  6  +-----+           |                                   *****
  -0 321100                  6  *-----*           |                                ****
  -0 99887665555            11  |  +  |           |                           *****+
  -1 4421100                 7  +-----+      -1.25+                       *****++
  -1 975                     3     |              |                    ***+++
  -2 4                       1     |              |                   *+++
  -2 8                       1     |              |                 *++
  -3                                              |              +++
  -3 7                       1     0              |           +++ *
  -4 3                       1     0              | +++    *
  -4                                              |+
  -5                                              |
  -5                                              |
  -6                                              |
  -6 7                       1     *         -6.75+   *
     ----+----+----+----+                          +----+----+----+----+----+----+----+----+----+----+
                                                        -2        -1         0        +1        +2
```

Figure 27. SAS Univariate summary of distribution of percent difference for NAC (PCNAC) from Elec-AC sample: RPRISM vs. PRISM

```
VARIABLE=PCBHO

                 MOMENTS                                        QUANTILES(DEF=4)                      EXTREMES

N                 50  SUM WGTS           50  100% MAX    18.0015      99%     18.0015     LOWEST     HIGHEST
MEAN         1.44984  SUM           72.4922   75% Q3     2.30829      95%     15.1733   -9.42339     4.89031
STD DEV       4.6915  VARIANCE      22.0102   50% MED    0.86952      90%     4.80344   -6.99705     7.51556
SKEWNESS     1.69969  KURTOSIS      5.62012   25% Q1   -0.171053      10%    -2.25195   -4.59451      13.194
USS           1183.6  CSS            1078.5    0% MIN   -9.42339       5%    -5.67565   -3.22871     17.5926
CV           323.587  STD MEAN     0.663479                            1%    -9.42339    -2.2766     18.0015
T:MEAN=0     2.18521  PROB>|T|    0.0336836  RANGE      27.4249
SGN RANK       268.5  PROB>|S|   0.00767882  Q3-Q1      2.47935
NUM ¬= 0          49                         MODE      -9.42339

STEM LEAF                    #  BOXPLOT                          NORMAL PROBABILITY PLOT
  18 0                       1     *            19+                                                    *
  16 6                       1     *              |                                                *
  14                                              |
  12 2                       1     *              |                                              *     +++
  10                                              |                                                ++++
   8                                              |                                           +++++
   6 5                         1     0            |                                       ++++ *
   4 09                        2     |           5+                                   ++++  **
   2 1113480068               10  +-----+         |                               +++*********
   0 11122448999004789        17  *--+--*         |                          *********
  -0 99955221100              11  +-----+         |                   ********+
  -2 230                       3     |            |               * **++++
  -4 6                         1     0            |              *++++
  -6 0                         1     0            |          ++*++
  -8 4                         1     *          -9+ ++*+
     ----+----+----+----+                          +----+----+----+----+----+----+----+----+----+----+
                                                       -2        -1         0        +1        +2
```

Figure 28. SAS Univariate summary of distribution of percent difference for βH_o (PCBHO) from Elec-AC sample: RPRISM vs. PRISM

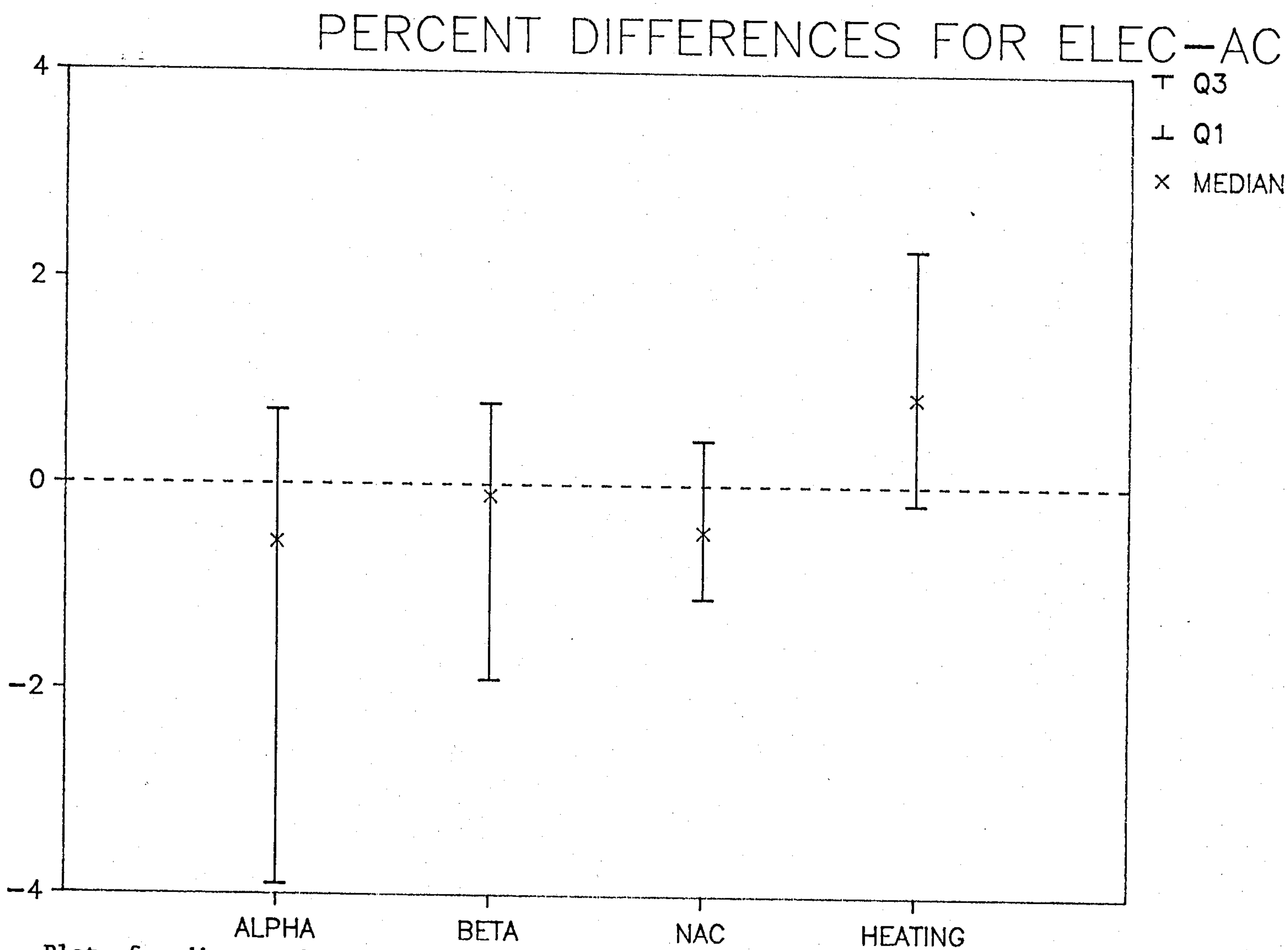


Figure 29. Plot of medians and quartiles of percent differences of PRISM vs. RPRISM parameters for Elec-AC sample.

```
VARIABLE=CVRALPHA

              MOMENTS                                   QUANTILES(DEF=4)                        EXTREMES

N                 50  SUM WGTS          50  100% MAX    2.0941       99%      2.0941       LOWEST     HIGHEST
MEAN         1.14854  SUM          57.4268   75% Q3    1.29692       95%     1.75446     0.761248     1.53914
STD DEV     0.275786  VARIANCE    0.076058   50% MED   1.07846       90%     1.53312     0.837273     1.62985
SKEWNESS     1.36573  KURTOSIS     2.00175   25% Q1   0.934605       10%    0.892262     0.849952     1.68528
USS          69.6837  CSS          3.72684    0% MIN  0.761248        5%    0.844246     0.888126     1.83901
CV            24.012  STD MEAN    0.039002                            1%    0.761248     0.891383      2.0941
T:MEAN=0     29.4481  PROB>|T|      0.0001  RANGE      1.33285
SGN RANK       637.5  PROB>|S|      0.0001  Q3-Q1     0.362314
NUM ¬= 0          50                        MODE      0.761248

STEM LEAF                   #  BOXPLOT                     NORMAL PROBABILITY PLOT
  20 9                      1     0       2.05+                                                    *
  19                                          |
  18 4                      1     |           |                                                *     ++
  17                                |           |                                                  ++++
  16 39                     2     |           |                                              **+++
  15 4                      1     |           |                                            *+++
  14 2448                   4     |           |                                         ****
  13 159                    3     |           |                                       +***
  12 0259                   4  +-----+        |                                   +++++**
  11 33345667               8  |  +  |        |                                ++*****
  10 00023488               8  *-----*        |                            ++****
   9 0033333446788         13  +-----+        |                     **********
   8 4599                   4     |           |               * **  *+++
   7 6                      1     |       0.75+    *        . ++++
     ----+----+----+----+                      +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**-01                     -2        -1         0        +1        +2
```

Figure 30. SAS Univariate summary of distribution of CVRs for α (CVRALPHA) from Elec-AC sample.

```
VARIABLE=CVRBETA

                 MOMENTS                                      QUANTILES(DEF=4)                        EXTREMES

N                  50  SUM WGTS         50  100% MAX   2.49047      99%      2.49047      LOWEST     HIGHEST
MEAN          1.21077  SUM         60.5383   75% Q3    1.35675      95%      2.28764    0.748206     1.56902
STD DEV       0.38262  VARIANCE   0.146398   50% MED   1.05956      90%      1.56867    0.826666     1.93601
SKEWNESS      1.84822  KURTOSIS    3.56017   25% Q1   0.968688      10%     0.916948    0.831043     2.16878
USS           80.4711  CSS          7.1735    0% MIN  0.748206       5%     0.829073    0.899257     2.43291
CV            31.6015  STD MEAN  0.0541106                           1%     0.748206      0.9158     2.49047
T:MEAN=0      22.3757  PROB>|T|     0.0001  RANGE      1.74226
SGN RANK        637.5  PROB>|S|     0.0001  Q3-Q1     0.388065
NUM ¬= 0           50                       MODE      0.748206

STEM LEAF                   #  BOXPLOT                     NORMAL PROBABILITY PLOT
  24 39                     2     0       2.45+                                                  *   *
  23                                          |
  22                                          |
  21 7                      1     0           |                                                *         ++
  20                                          |                                                        +++
  19 4                      1     |           |                                               *   ++
  18                                          |                                                  +++
  17                                          |                                               +++
  16                                          |                                             ++
  15 03577                  5     |           |                                          ++****
  14 07                     2     |           |                                       +++ *
  13 02558                  5  +-----+        |                                     ++****
  12 78                     2  |  +  |        |                                  +++ *
  11 1278                   4  |     |        |                               ++  **
  10 000013335789          12  *-----*        |                           +******
   9 0233344448999         13  +-----+        |                  **********
   8 33                     2     |           |         * **      ++
   7 5                      1     |       0.75+      *        +++
     ----+----+----+----+                      +----+----+----+----+----+----+----+----+----+----+
   MULTIPLY STEM.LEAF BY 10**-01                    -2        -1         0        +1        +2
```

Figure 31. SAS Univariate summary of distribution of CVRs for β (CVRBETA) from Elec-AC sample.

```
VARIABLE=CVRNAC

               MOMENTS                                         QUANTILES(DEF=4)                        EXTREMES

N                  50  SUM WGTS           50  100% MAX    2.29004      99%    2.29004     LOWEST     HIGHEST
MEAN          1.18269  SUM           59.1345   75% Q3      1.3337      95%    1.97893   0.757609     1.57713
STD DEV      0.320822  VARIANCE     0.102927   50% MED    1.08303      90%    1.57521   0.828821      1.6888
SKEWNESS      1.56515  KURTOSIS      2.62738   25% Q1    0.939813      10%   0.926884   0.845201     1.89823
USS           74.9812  CSS           5.04341    0% MIN   0.757609       5%    0.83783   0.899719     2.07757
CV            27.1265  STD MEAN    0.0453711                            1%   0.757609   0.926798     2.29004
T:MEAN=0       26.067  PROB>|T|       0.0001  RANGE       1.53243
SGN RANK        637.5  PROB>|S|       0.0001  Q3-Q1      0.393883
NUM ¬= 0           50                         MODE       0.757609

STEM LEAF                    #  BOXPLOT                          NORMAL PROBABILITY PLOT
  22 9                       1     0        2.25+                                                    *
  21                                            |
  20 8                       1     0            |                                                *
  19 0                       1     |        1.95+                                                     +++
  18                                 |            |                                              *    +++
  17                                 |            |                                               +++
  16 9                       1     |        1.65+                                            +*+
  15 00168                   5     |            |                                        ****
  14 03                      2     |            |                                     ++*
  13 19                      2  +-----+     1.35+                                  +++**
  12 3579                    4  |     |         |                               +++***
  11 025578                  6  |  +  |         |                            +++ ***
  10 0022467889             10  *-----*     1.05+                         +******
   9 03333334446779         14  +-----+         |              **********
   8 35                      2     |            |       * **   +++
   7 6                       1     |        0.75+    *     ++++
     ----+----+----+----+                       +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**-01                         -2        -1         0        +1        +2
```

Figure 32. SAS Univariate summary of distribution of CVRs for NAC (CVRNAC) from Elec-AC sample.

```
VARIABLE=CVRBHO

                MOMENTS                                            QUANTILES(DEF=4)                          EXTREMES

N              50  SUM WGTS          50  100% MAX   2.66135     99%     2.66134     LOWEST    HIGHEST
MEAN      1.20309  SUM          60.1543   75% Q3    1.32039     95%     2.06886   0.750756    1.64611
STD DEV  0.365876  VARIANCE    0.133865   50% MED   1.07567     90%     1.64172   0.819553     1.6931
SKEWNESS  2.01373  KURTOSIS     5.11778   25% Q1   0.946623     10%    0.918417   0.842051    1.88937
USS       78.9301  CSS           6.5594    0% MIN  0.750756      5%    0.831927   0.900559    2.28823
CV        30.4115  STD MEAN   0.0517427                          1%    0.750756   0.918009    2.66135
T:MEAN=0  23.2513  PROB>|T|      0.0001  RANGE      1.91059
SGN RANK    637.5  PROB>|S|      0.0001  Q3-Q1      0.37377
NUM ^= 0       50                        MODE      0.750756

STEM LEAF                  #  BOXPLOT                          NORMAL PROBABILITY PLOT
  26 6                     1     *          2.65+                                                    *
  25                                            |
  24                                            |
  23                                            |
  22 9                     1     0              |                                                *
  21                                            |                                                     +
  20                                            |                                                  +++
  19                                            |                                                ++
  18 9                     1     0              |                                             *+++
  17                                 |          |                                           +++
  16 059                   3         |          |                                        +** *
  15 069                   3         |          |                                      +**
  14 2                     1         |          |                                   +++*
  13 0078                  4  +-----+           |                                 +++ **
  12 2399                  4  |  +  |           |                              +++ ***
  11 025667                6  |     |           |                            ++ ***
  10 00123336778          11  *-----*           |                         +*****
   9 022333444568         12  +-----+           |               * *********
   8 24                    2     |              |            * *    +++
   7 5                     1     |          0.75+   *             ++
     ----+----+----+----+                       +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**-01                      -2        -1         0        +1        +2
```

Figure 33. SAS Univariate summary of distribution of CVRs for βH_o (CVRBHO) from Elec-AC sample.

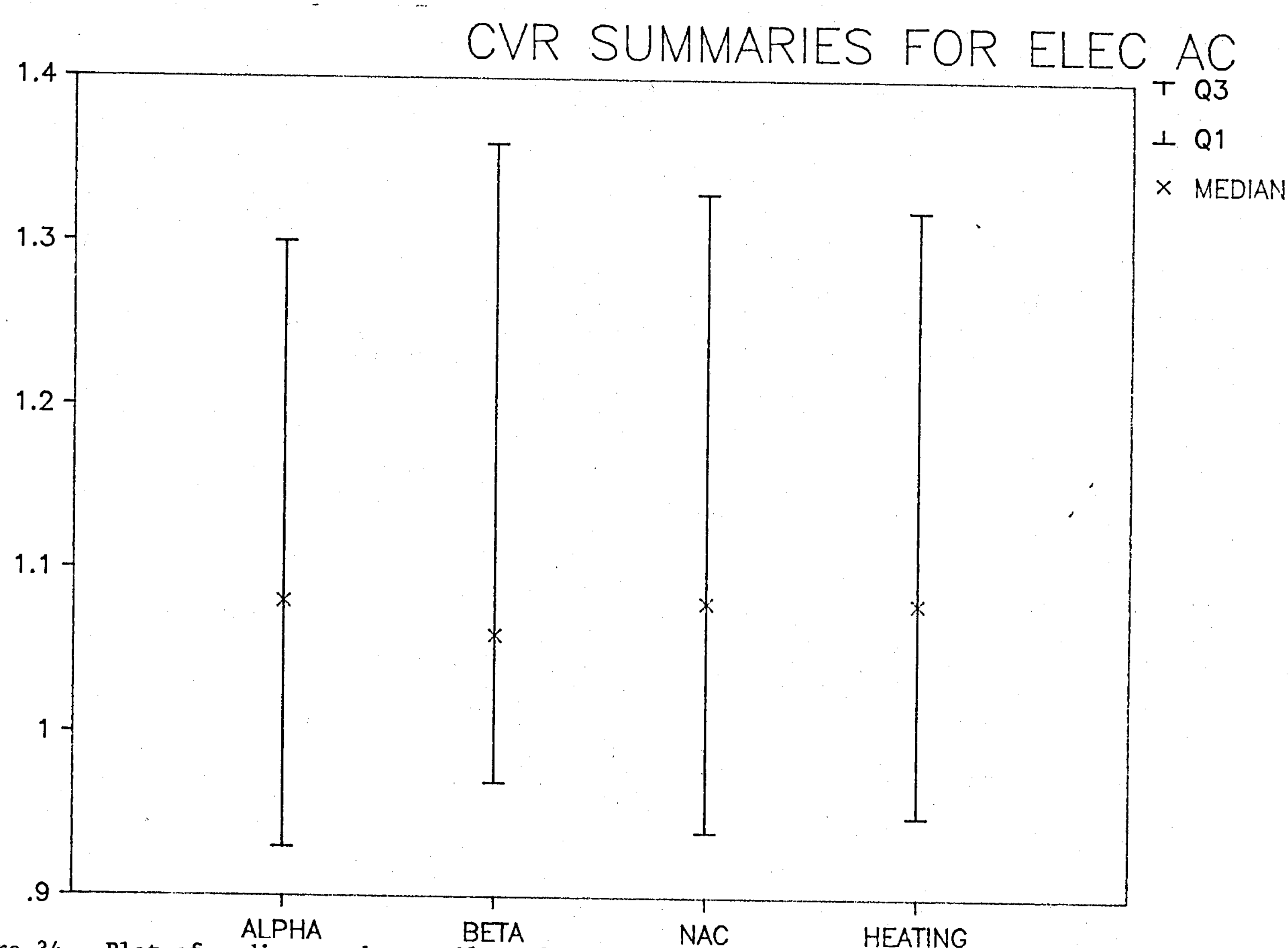


Figure 34. Plot of medians and quartiles of CVRs of PRISM vs. RPRISM parameters for Elec-AC sample.

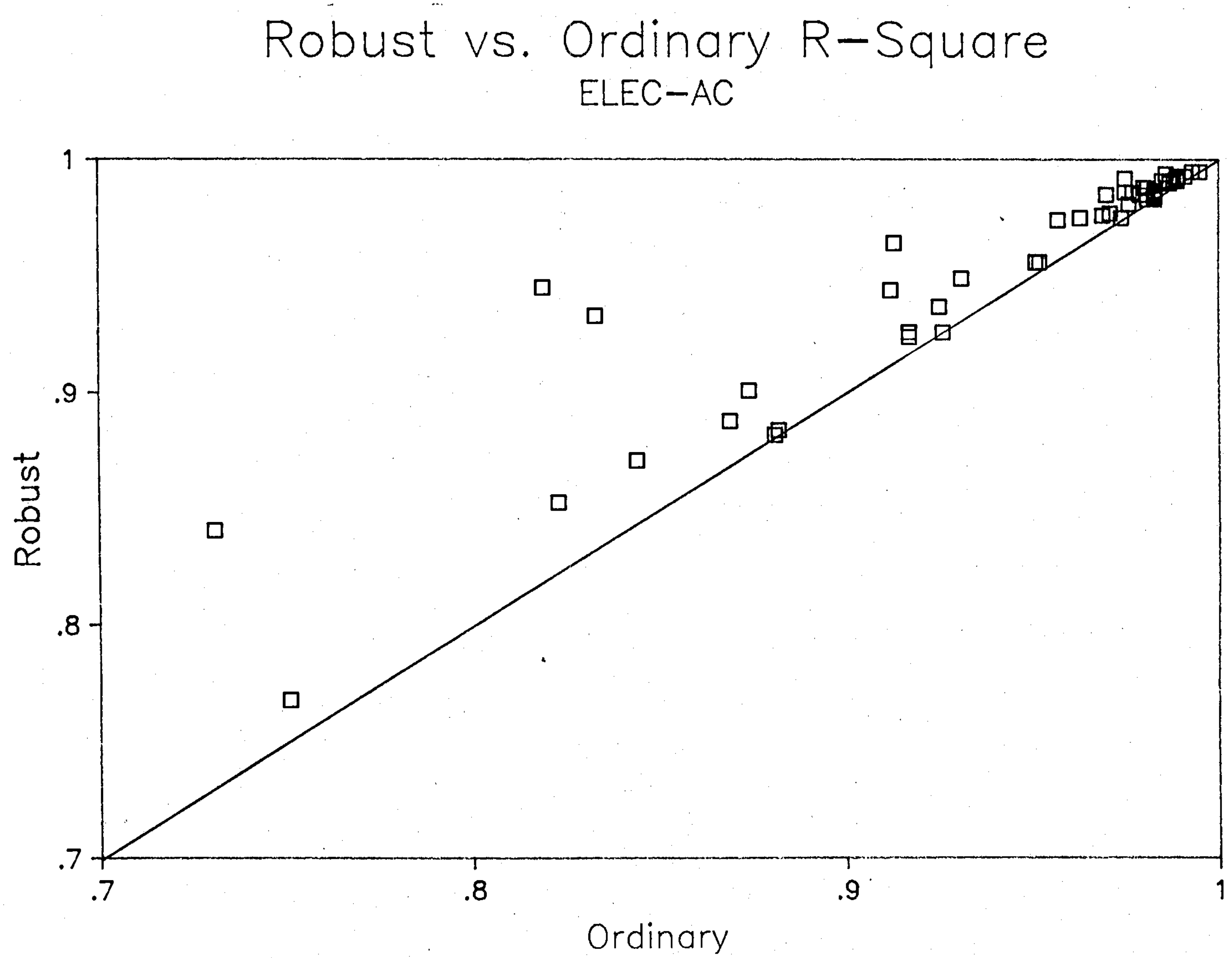
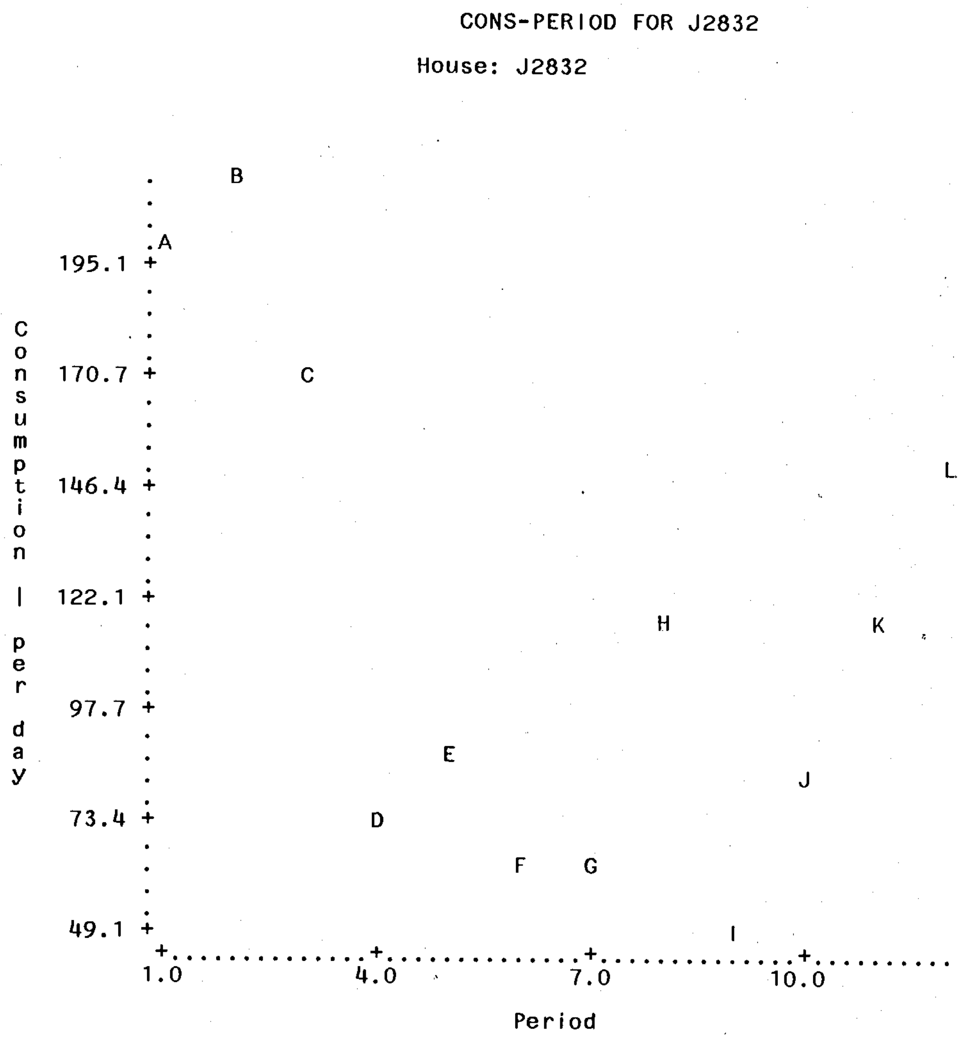


Figure 35. R^2 estimates for Robust (RPRISM) vs. Ordinary PRISM for Elec-AC sample.



PERIODS:

A DEC 26,1978 to JAN 24,1979
B JAN 25,1979 to FEB 26,1979
C FEB 27,1979 to MAR 26,1979
D MAR 27,1979 to APR 25,1979
E APR 26,1979 to MAY 24,1979
F MAY 25,1979 to JUN 25,1979
G JUN 26,1979 to JUL 25,1979
H JUL 26,1979 to AUG 23,1979
I AUG 24,1979 to SEP 24,1979
J SEP 25,1979 to OCT 24,1979
K OCT 25,1979 to NOV 25,1979
L NOV 26,1979 to DEC 25,1979

Figure 36. Plot of consumption vs. period for House J2832 from Elec-AC sample.

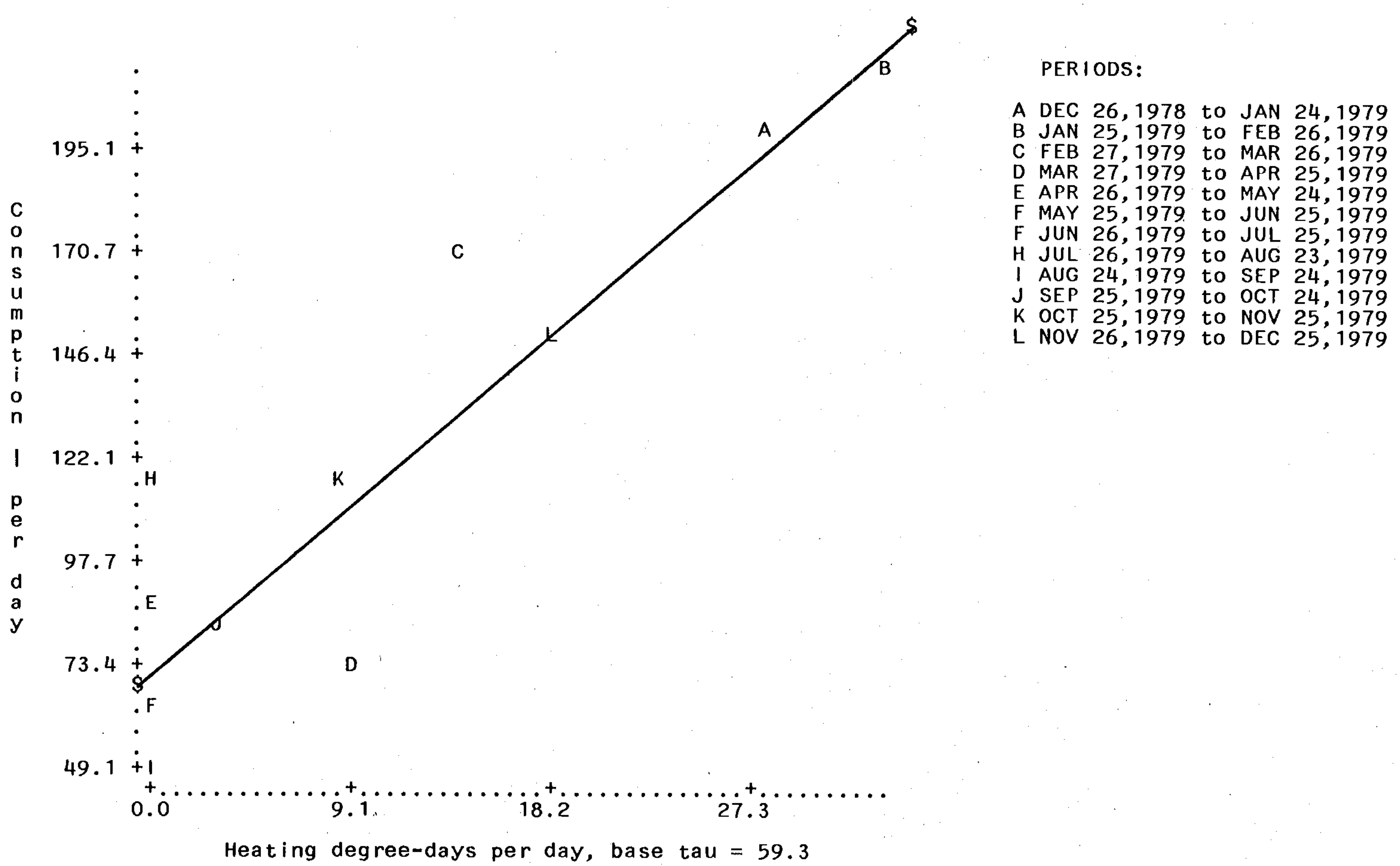
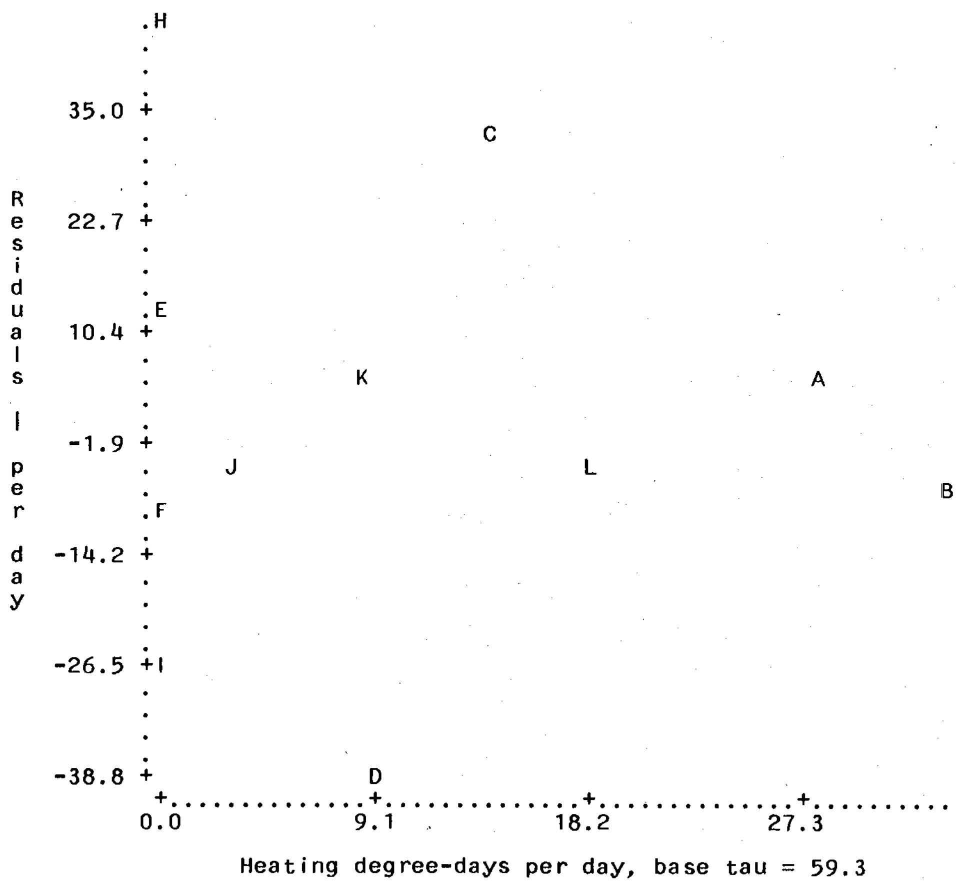


Figure 37. PRISM plot of consumption vs. heating degree-days for House J2832 from Elec-AC sample.

RESIDS-HDD FOR J2832, PRISM



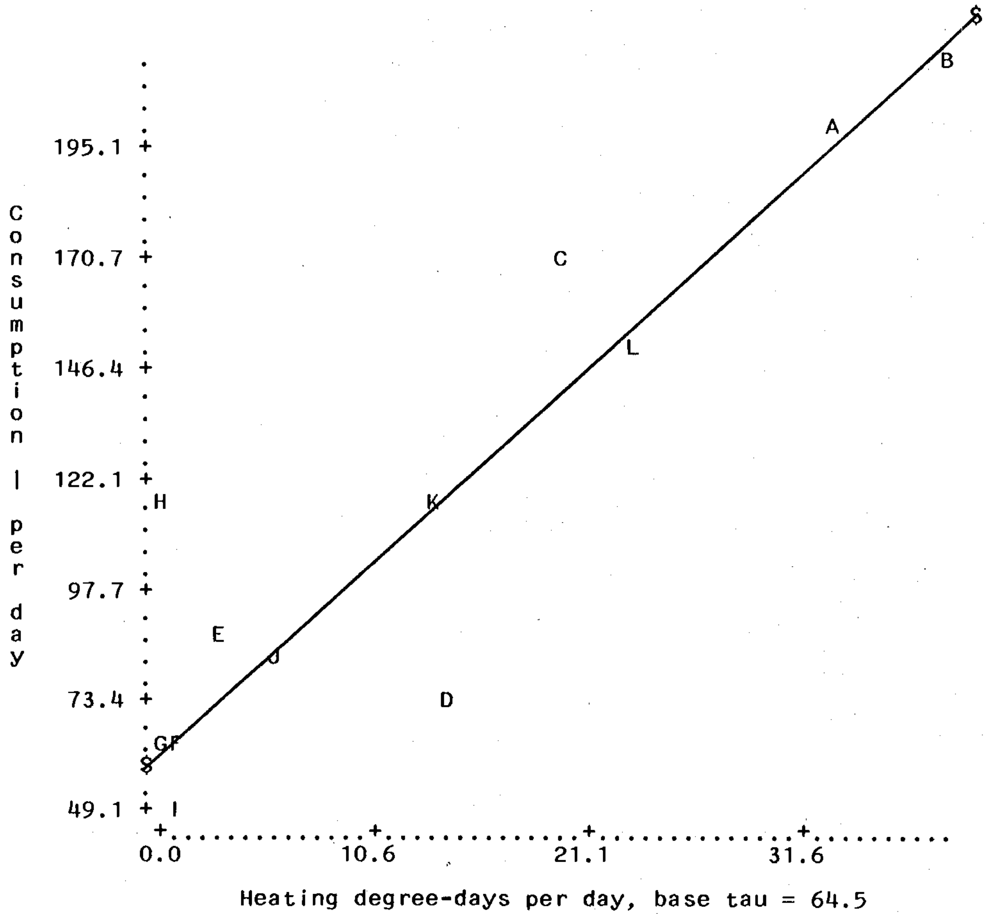
PERIODS:

A DEC 26,1978 to JAN 24,1979
B JAN 25,1979 to FEB 26,1979
C FEB 27,1979 to MAR 26,1979
D MAR 27,1979 to APR 25,1979
E APR 26,1979 to MAY 24,1979
F MAY 25,1979 to JUN 25,1979
F JUN 26,1979 to JUL 25,1979
H JUL 26,1979 to AUG 23,1979
I AUG 24,1979 to SEP 24,1979
J SEP 25,1979 to OCT 24,1979
K OCT 25,1979 to NOV 25,1979
L NOV 26,1979 to DEC 25,1979

Figure 38. PRISM plot of residuals vs. heating degree-days for House J2832 from Elec-AC sample.

CONS-HDD FOR J2832, RPRISM

House:J2832 ,alpha= 64.47,beta= 3.95,R2= 0.9330

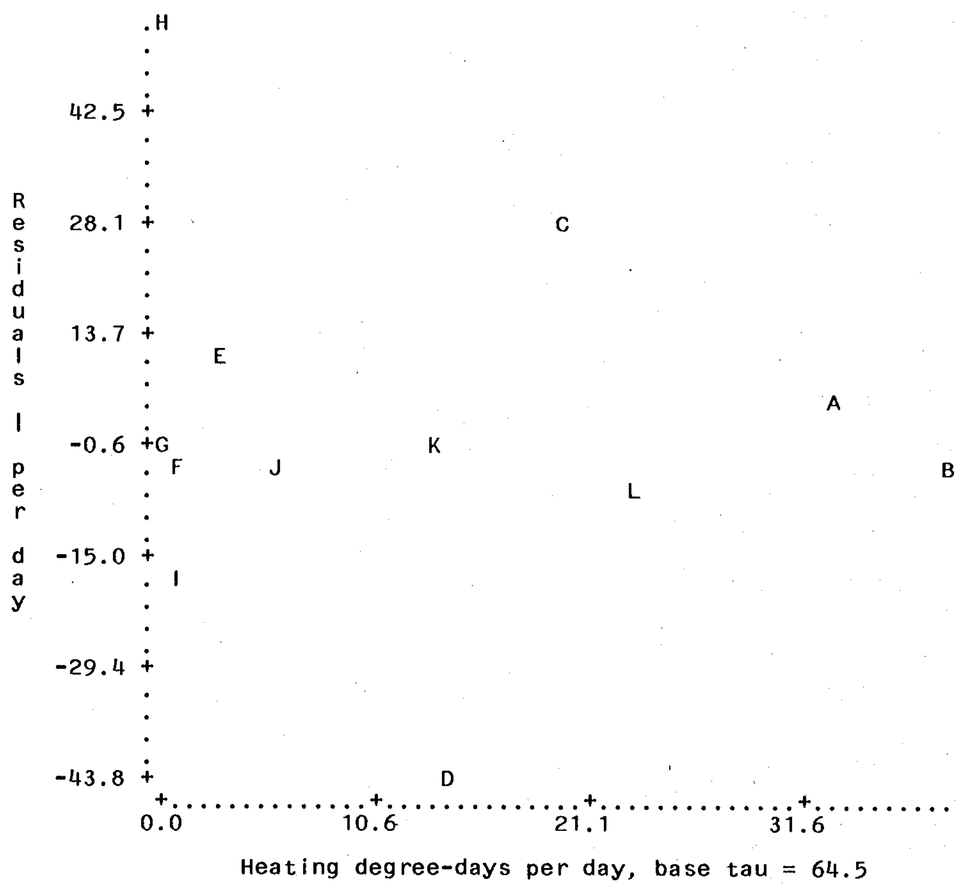


PERIODS:

A DEC 26,1978 to JAN 24,1979
B JAN 25,1979 to FEB 26,1979
C FEB 27,1979 to MAR 26,1979
D MAR 27,1979 to APR 25,1979
E APR 26,1979 to MAY 24,1979
F MAY 25,1979 to JUN 25,1979
G JUN 26,1979 to JUL 25,1979
H JUL 26,1979 to AUG 23,1979
I AUG 24,1979 to SEP 24,1979
J SEP 25,1979 to OCT 24,1979
K OCT 25,1979 to NOV 25,1979
L NOV 26,1979 to DEC 25,1979

Figure 39. RPRISM plot of consumption vs. heating degree-days for House J2832 from Elec-AC sample.

RESIDS-HDD FOR J2832, RPRISM



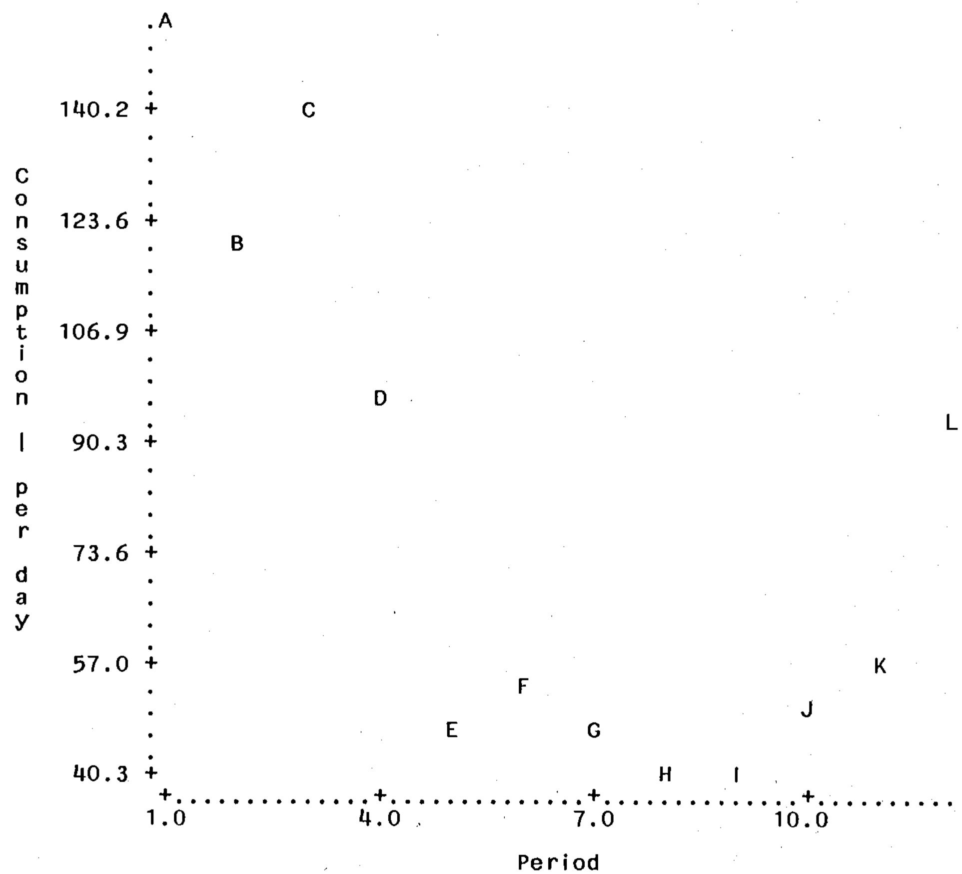
PERIODS:

A DEC 26,1978 to JAN 24,1979
B JAN 25,1979 to FEB 26,1979
C FEB 27,1979 to MAR 26,1979
D MAR 27,1979 to APR 25,1979
E APR 26,1979 to MAY 24,1979
F MAY 25,1979 to JUN 25,1979
G JUN 26,1979 to JUL 25,1979
H JUL 26,1979 to AUG 23,1979
I AUG 24,1979 to SEP 24,1979
J SEP 25,1979 to OCT 24,1979
K OCT 25,1979 to NOV 25,1979
L NOV 26,1979 to DEC 25,1979

Figure 40. RPRISM plot of residuals vs. heating degree-days for House J2832 from Elec-AC sample.

CONS-PERIOD FOR J3075

House: J3075



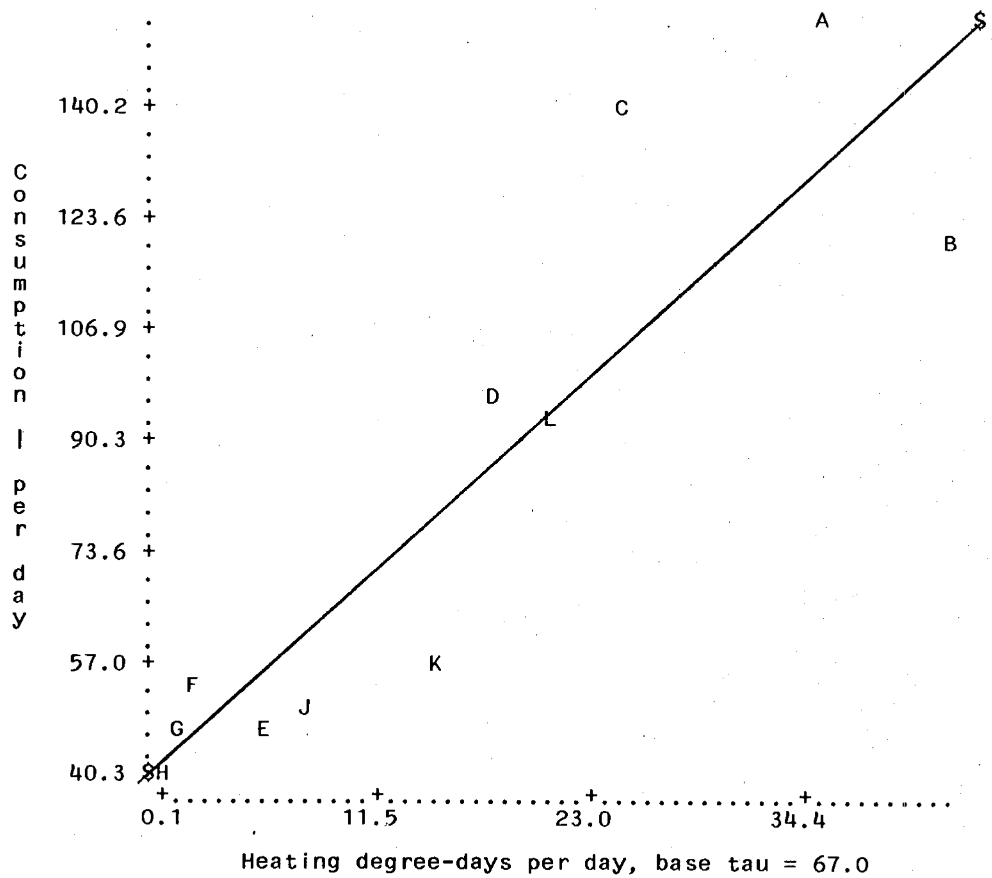
PERIODS:

A DEC 19,1978 to JAN 18,1979
B JAN 19,1979 to FEB 20,1979
C FEB 21,1979 to MAR 20,1979
D MAR 21,1979 to APR 19,1979
E APR 20,1979 to MAY 20,1979
F MAY 21,1979 to JUN 19,1979
G JUN 20,1979 to JUL 19,1979
H JUL 20,1979 to AUG 19,1979
I AUG 20,1979 to SEP 18,1979
J SEP 19,1979 to OCT 18,1979
K OCT 19,1979 to NOV 18,1979
L NOV 19,1979 to DEC 18,1979

Figure 41. Plot of consumption vs. period for House J3075 from Elec-AC sample.

CONS-HDD FOR J3075, PRISM

House:J3075 ,alpha= 41.28,beta= 2.61,R2= 0.8233



PERIODS:

A DEC 19,1978 to JAN 18,1979
B JAN 19,1979 to FEB 20,1979
C FEB 21,1979 to MAR 20,1979
D MAR 21,1979 to APR 19,1979
E APR 20,1979 to MAY 20,1979
F MAY 21,1979 to JUN 19,1979
G JUN 20,1979 to JUL 19,1979
H JUL 20,1979 to AUG 19,1979
H AUG 20,1979 to SEP 18,1979
J SEP 19,1979 to OCT 18,1979
K OCT 19,1979 to NOV 18,1979
L NOV 19,1979 to DEC 18,1979

Figure 42. PRISM plot of consumption vs. heating-degree days for House J3075 from Elec-AC sample.

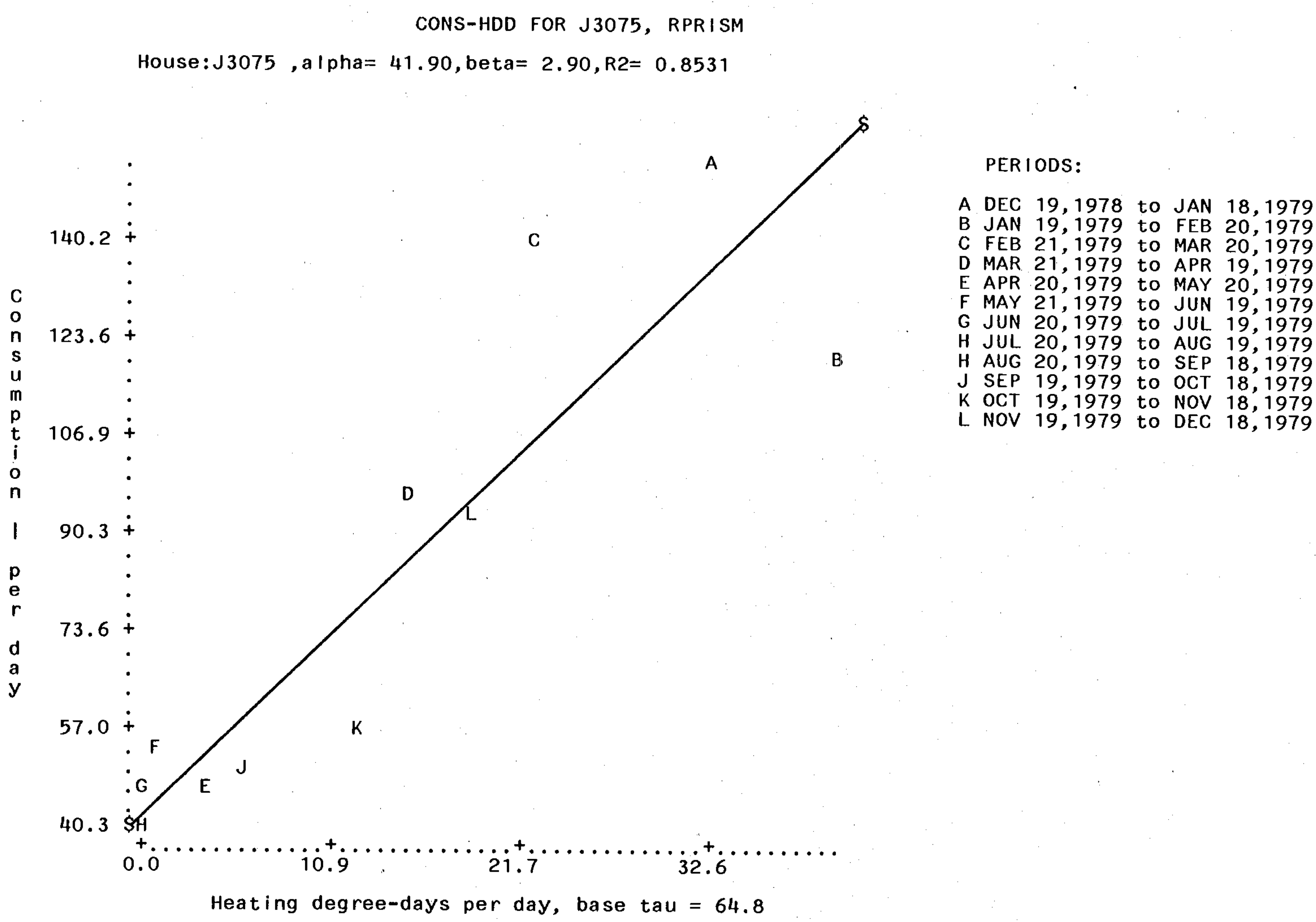
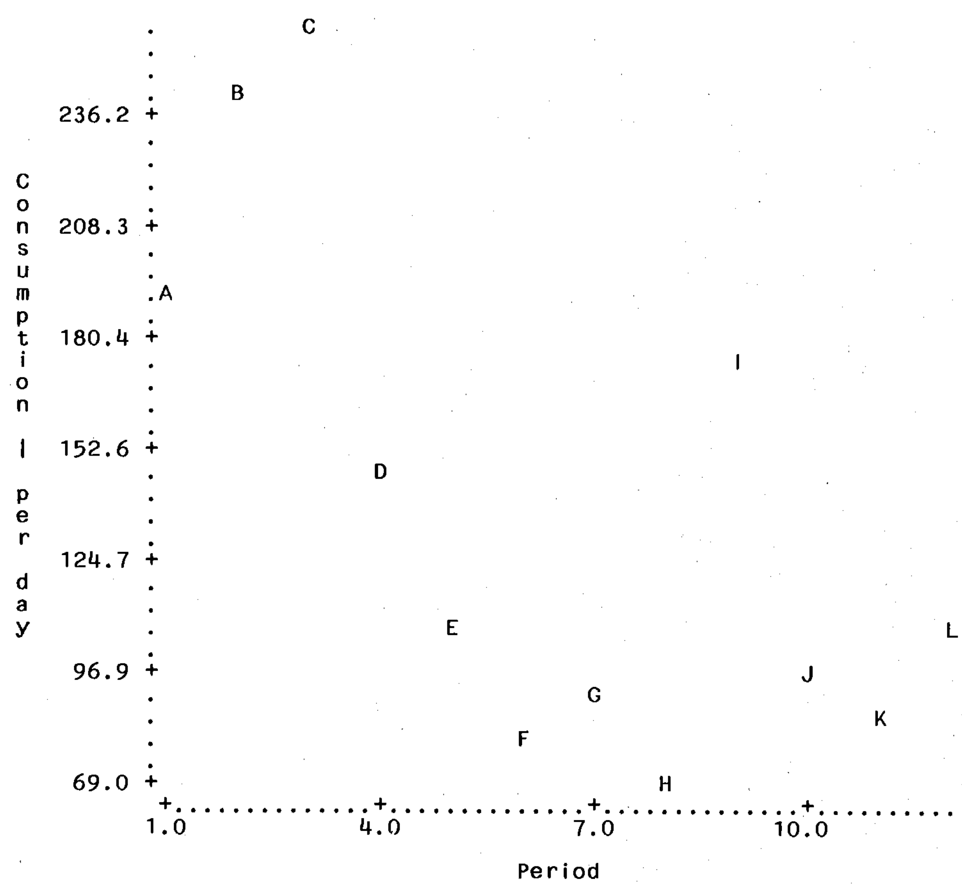


Figure 43. RPRISM plot of consumption vs. heating degree-days for House J3075 from Elec-AC sample.

CONS-PERIOD FOR J3076

House: J3076



PERIODS:

A	DEC	1,1978	to	JAN	2,1979
B	JAN	3,1979	to	JAN	31,1979
C	FEB	1,1979	to	MAR	4,1979
D	MAR	5,1979	to	APR	2,1979
E	APR	3,1979	to	MAY	2,1979
F	MAY	3,1979	to	JUN	3,1979
G	JUN	4,1979	to	JUL	2,1979
H	JUL	3,1979	to	AUG	1,1979
I	AUG	2,1979	to	AUG	30,1979
J	AUG	31,1979	to	OCT	1,1979
K	OCT	2,1979	to	OCT	31,1979
L	NOV	1,1979	to	DEC	2,1979

Figure 44. Plot of consumption vs. period for House J3076 from Elec-AC sample.

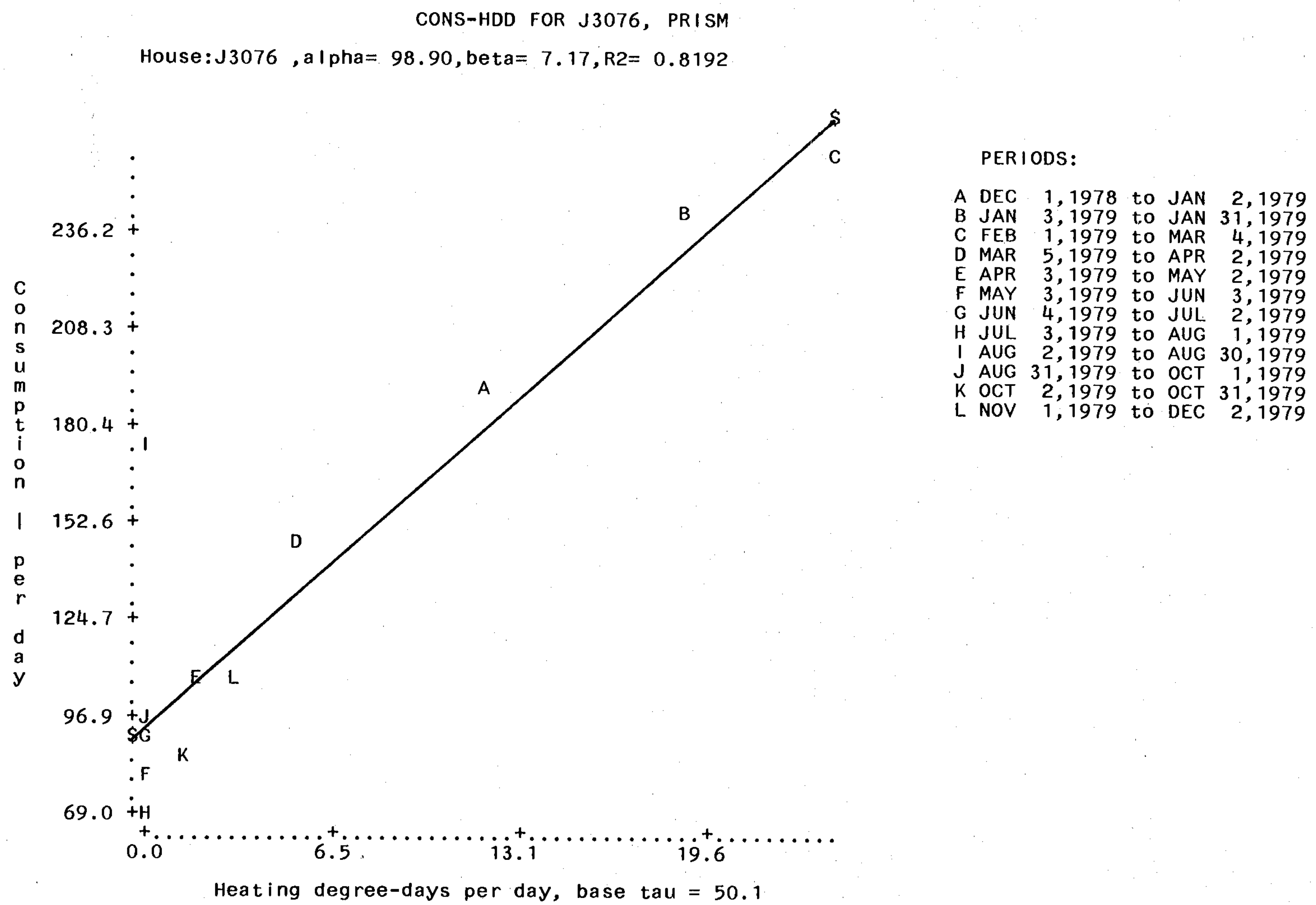
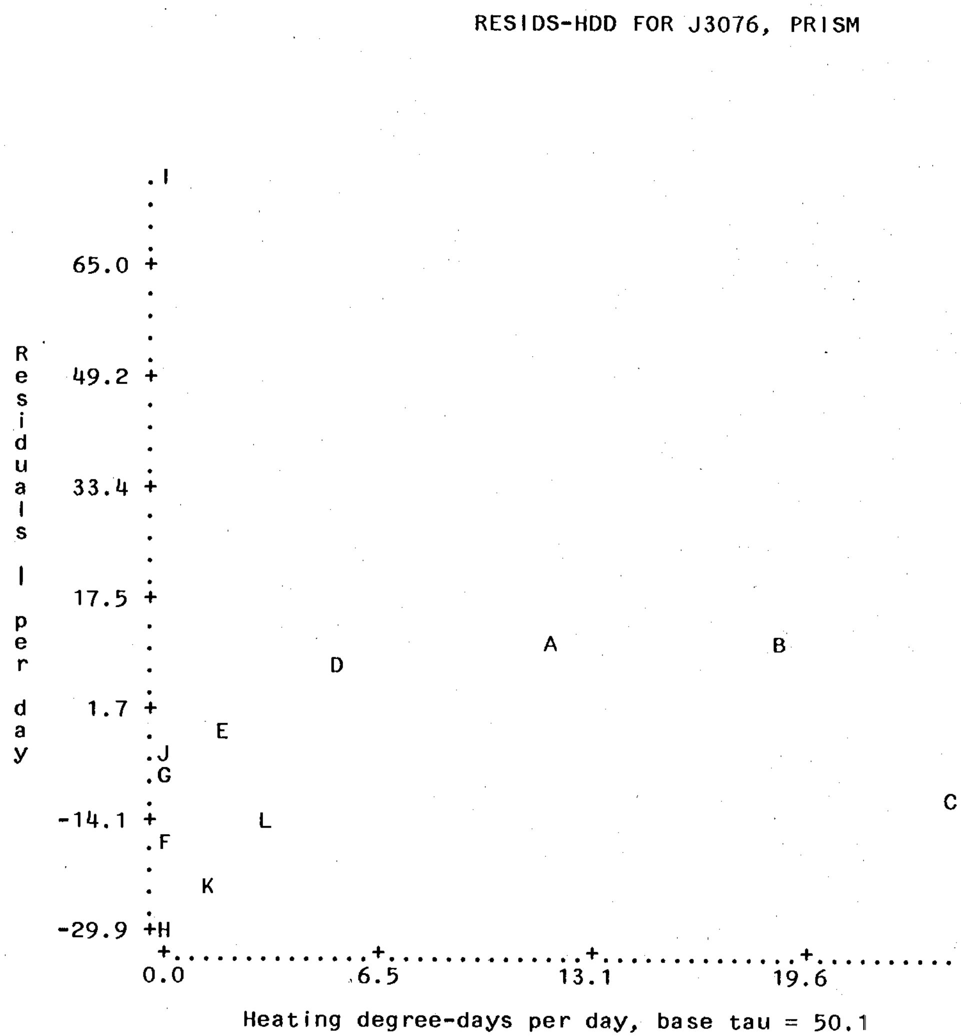


Figure 45. PRISM plot of consumption vs. heating degree-days for House J3076 from Elec-AC sample.



PERIODS:

A	DEC 1,1978	to	JAN 2,1979
B	JAN 3,1979	to	JAN 31,1979
C	FEB 1,1979	to	MAR 4,1979
D	MAR 5,1979	to	APR 2,1979
E	APR 3,1979	to	MAY 2,1979
F	MAY 3,1979	to	JUN 3,1979
G	JUN 4,1979	to	JUL 2,1979
H	JUL 3,1979	to	AUG 1,1979
I	AUG 2,1979	to	AUG 30,1979
J	AUG 31,1979	to	OCT 1,1979
K	OCT 2,1979	to	OCT 31,1979
L	NOV 1,1979	to	DEC 2,1979

Figure 46. PRISM plot of residuals vs. heating degree-days for House J3076 from Elec-AC sample.

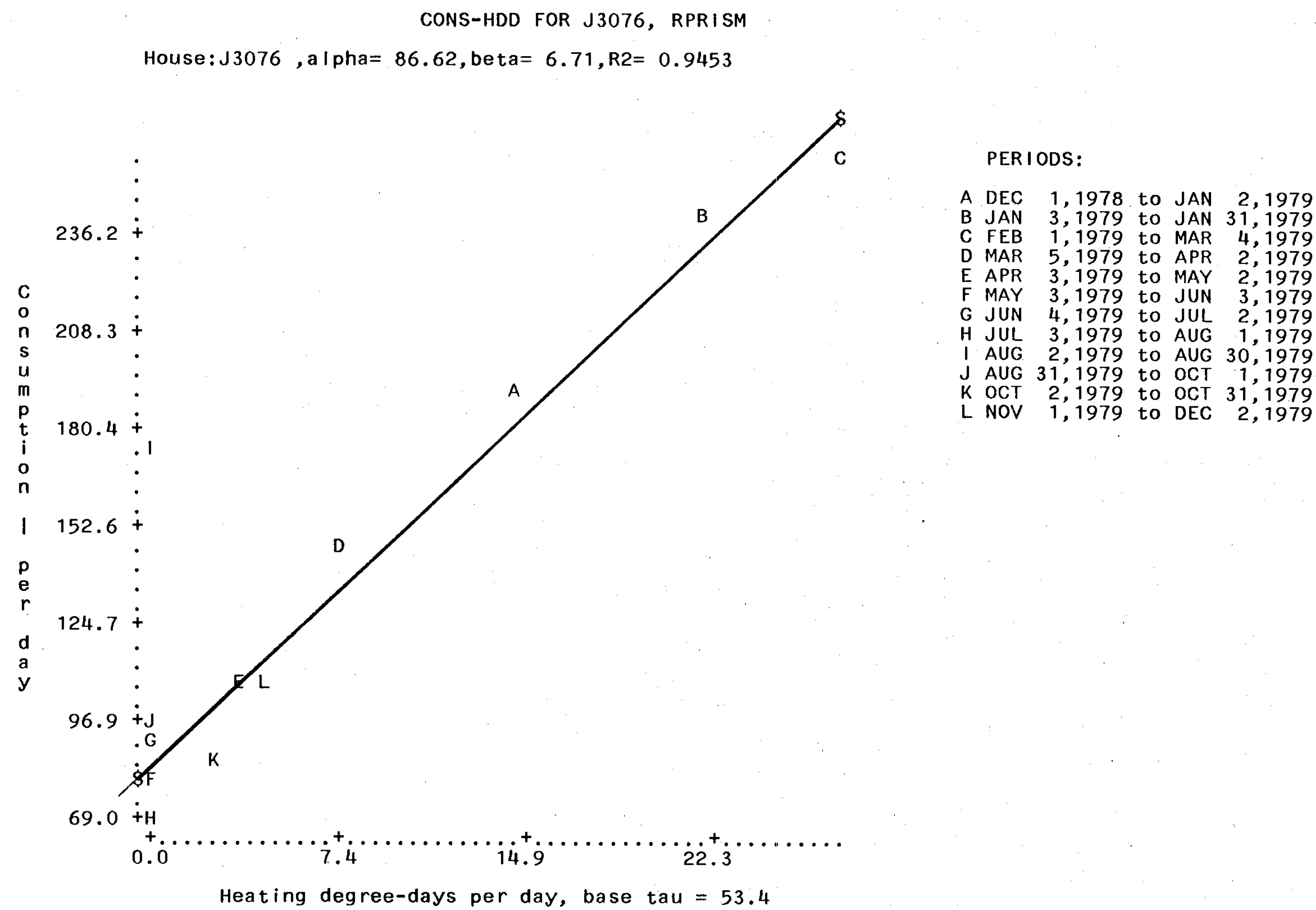
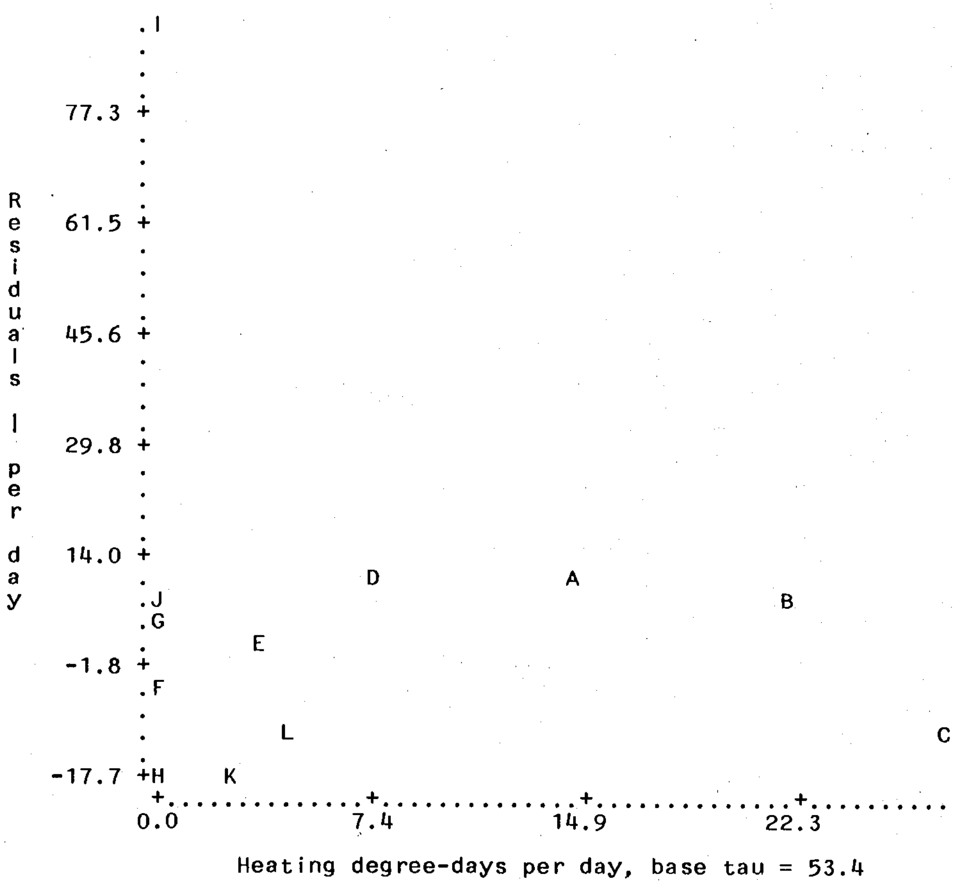


Figure 47. RPRISM plot of consumption vs. heating degree-days for House J3076 from Elec-AC sample.

RESIDS-HDD FOR J3076, RPRISM



PERIODS:

A DEC 1,1978 to JAN 2,1979
B JAN 3,1979 to JAN 31,1979
C FEB 1,1979 to MAR 4,1979
D MAR 5,1979 to APR 2,1979
E APR 3,1979 to MAY 2,1979
F MAY 3,1979 to JUN 3,1979
G JUN 4,1979 to JUL 2,1979
H JUL 3,1979 to AUG 1,1979
I AUG 2,1979 to AUG 30,1979
J AUG 31,1979 to OCT 1,1979
K OCT 2,1979 to OCT 31,1979
L NOV 1,1979 to DEC 2,1979

Figure 48. RPRISM plot of residuals vs. heating degree-days for House J3076 from Elec-AC sample.

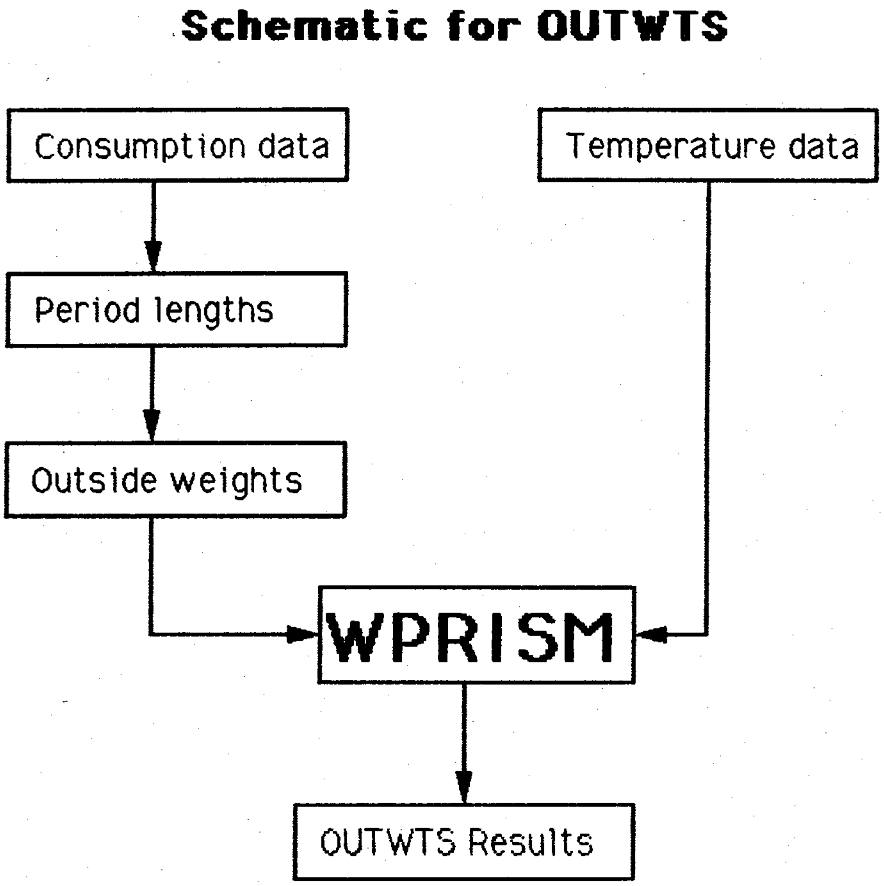


Figure 49. Schematic of weighted PRISM (OUTWTS) algorithm.

```
VARIABLE=PCALPHA

                 MOMENTS                                      QUANTILES(DEF=4)                        EXTREMES

N                 69  SUM WGTS           69  100% MAX      2150       99%       2150       LOWEST    HIGHEST
MEAN         42.5194  SUM           2933.84   75% Q3    23.1777       95%    317.855     -153.846        100
STD DEV      270.934  VARIANCE      73405.1   50% MED  -1.26582       90%    84.2102         -125        250
SKEWNESS     7.18082  KURTOSIS      55.7748   25% Q1   -24.1902       10%   -50.6682          -75    385.714
USS          5116291  CSS           4991545    0% MIN  -153.846        5%   -70.8333     -66.6667        400
CV             637.2  STD MEAN      32.6166                            1%   -153.846        -62.5       2150
T:MEAN=0     1.30361  PROB>|T|     0.196761   RANGE     2303.85
SGN RANK         -11  PROB>|S|     0.949942   Q3-Q1     47.3679
NUM ¬= 0          69                          MODE     -153.846

STEM LEAF                                         #  BOXPLOT                     NORMAL PROBABILITY PLOT
  21 5                                            1     *         2150+                                                        *
  20                                                                  |
  19                                                                  |
  18                                                                  |
  17                                                                  |
  16                                                                  |
  15                                                                  |
  14                                                                  |
  13                                                                  |
  12                                                                  |
  11                                                                  |
  10                                                                  |
   9                                                                  |
   8                                                                  |
   7                                                                  |                                                         +
   6                                                                  |                                                     ++++
   5                                                                  |                                                 ++++
   4 0                                            1                   |                                             +++
   3 9                                            1     *             |                                         ++++    * *
   2 5                                            1     *             |                                     ++++       *
   1 00                                           2                   |                                  +++
   0 111111122223335556668                       21  +--0--+          |                              ++****************
  -0 7766554433333222222221111111100000000000    40  *-----*          |      * *****************
  -1 52                                           2     0        -150+ *   *           +++
     ----+----+----+----+----+----+----+----+                         +----+----+----+----+----+----+----+----+----+----+
   MULTIPLY STEM.LEAF BY 10**+02                                           -2        -1         0        +1        +2
```

Figure 50. SAS Univariate summary of distribution of percent difference for α (PCALPHA) from oil sample: OUTWTS vs. PRISM

```
VARIABLE=PCBETA

                  MOMENTS                                           QUANTILES(DEF=4)                        EXTREMES

N                 69  SUM WGTS            69  100% MAX    10.5023     99%     10.5023      LOWEST     HIGHEST
MEAN         -1.8966  SUM           -130.865   75% Q3     2.69058     95%     7.22295    -51.2821     6.53266
STD DEV      11.2099  VARIANCE       125.661   50% MED   0.416667     90%     5.10947    -44.6927     6.64452
SKEWNESS    -3.33988  KURTOSIS       11.2992   25% Q1    -2.32646     10%    -4.93273    -44.3548     7.80142
USS          8793.13  CSS            8544.93    0% MIN   -51.2821      5%    -40.7873    -37.2197     10.2804
CV          -591.051  STD MEAN       1.34951                           1%     -51.282    -5.11945     10.5023
T:MEAN=0     -1.4054  PROB>|T|      0.164455  RANGE       61.7843
SGN RANK          75  PROB>|S|      0.641662  Q3-Q1       5.01704
NUM ¬= 0          67                          MODE              0

STEM LEAF                              #  BOXPLOT                          NORMAL PROBABILITY PLOT
   1 01                                2     0        12.5+                                   +++++   *   *
   0 56778                             5     |            |                               ++++  **** *
   0 11111111222222233333444444       26  *-----*         |                       *************
  -0 4444333332222221111100000000     28  +--+--+         |               **************++
  -0 5555                              4     |            |             **     +++++
  -1                                                      |                 ++++
  -1                                                      |            +++++
  -2                                                      |        ++++
  -2                                                      |+++++
  -3                                                      |
  -3 7                                 1     *            |            *
  -4 4                                 1     *            |      * *
  -4 5                                 1                  |
  -5 1                                 1     *       -52.5+ *
     ----+----+----+----+----+---                          +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**+01                                 -2        -1         0        +1        +2
```

Figure 51. SAS Univariate summary of distribution of percent difference for β (PCBETA) from OIL sample: OUTWTS vs. PRISM

```
VARIABLE=PCNAC

                    MOMENTS                                              QUANTILES(DEF=4)                        EXTREMES

N                  69  SUM WGTS            69 100% MAX    0.876062       99%     0.876062      LOWEST    HIGHEST
MEAN        -0.928853  SUM           -64.0908  75% Q3    -0.02277        95%     0.573579    -21.3673   0.474273
STD DEV       2.89981  VARIANCE       8.40891  50% MED  -0.384847        90%     0.337211    -10.7316   0.491111
SKEWNESS     -5.88844  KURTOSIS       38.6405  25% Q1   -0.763239        10%    -1.51826     -4.06362   0.656052
USS           631.337  CSS            571.806   0% MIN   -21.3673         5%    -3.51873     -2.97384   0.686807
CV           -312.193  STD MEAN      0.349096                             1%    -21.3673     -2.51574   0.876062
T:MEAN=0     -2.66073  PROB>|T|    0.00971847  RANGE      22.2433
SGN RANK       -829.5  PROB>|S|        0.0001  Q3-Q1     0.740469
NUM ¬= 0           69                          MODE      -21.3673

STEM LEAF                                          #  BOXPLOT                       NORMAL PROBABILITY PLOT
   0 222233455779                                 12     |            0.5+                         +++ ********* * *    *
  -0 99887777666555555544433322222221111110000000  44  *--+--*            |                  *****************
  -1 5543320                                       7     |                |               ****     +++
  -2 50                                            2     0                |            ***     ++++
  -3 0                                             1                      |                  +++
  -4 1                                             1     *                |          *   ++++
  -5                                                                      |           +++
  -6                                                                      |        +++
  -7                                                                      | ++++
  -8                                                                      |+
  -9                                                                      |
 -10 7                                             1     *          -10.5+     *
 -11                                                                      |
 -12                                                                      |
 -13                                                                      |
 -14                                                                      |
 -15                                                                      |
 -16                                                                      |
 -17                                                                      |
 -18                                                                      |
 -19                                                                      |
 -20                                                                      |
 -21 4                                             1     *          -21.5+ *
     ----+----+----+----+----+----+----+----+----                          +----+----+----+----+----+----+----+----+----+----+
                                                                                -2        -1         0        +1        +2
```

Figure 52. SAS Univariate summary of distribution of percent difference for NAC (PCNAC) from OIL sample: OUTWTS vs. PRISM

```
VARIABLE=PCBHO

              MOMENTS                                      QUANTILES(DEF=4)                         EXTREMES

N                69  SUM WGTS          69  100% MAX   680.306      99%     680.306      LOWEST    HIGHEST
MEAN         11.731  SUM          809.437   75% Q3    2.99092      95%     49.4463    -21.0807    32.4906
STD DEV     82.7649  VARIANCE     6850.03   50% MED  -0.35504      90%     8.41985    -13.5802    40.1319
SKEWNESS    7.98624  KURTOSIS     65.2666   25% Q1   -3.77335      10%    -6.70711    -8.78207    58.7613
USS          475297  CSS           465802    0% MIN  -21.0807       5%    -8.51878    -8.25549     68.446
CV          705.524  STD MEAN     9.96372                           1%    -21.0807    -8.03919    680.306
T:MEAN=0    1.17737  PROB>|T|    0.243153  RANGE      701.387
SGN RANK      -56.5  PROB>|S|     0.73776  Q3-Q1      6.76427
NUM ¬= 0         69                        MODE      -21.0807

STEM LEAF                                    #  BOXPLOT                       NORMAL PROBABILITY PLOT
   6                                         1     *         675+                                                *
   6                                                             |
   5                                                             |
   5                                                             |
   4                                                             |
   4                                                             |
   3                                                             |
   3                                                         325+
   2                                                             |
   2                                                             |                                             +++
   1                                                             |                                       ++++++
   1                                                             |                                 ++++++
   0                                         2     *             |                           ++++++      * *
   0                                        28  +--*--+          |                     ++****************
  -0                                        38  *--0--*      -25+ *   * * *****************
    ----+----+----+----+----+----+----+---                       +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**+02                                      -2        -1         0        +1        +2
```

Figure 53. SAS Univariate summary of distribution of percent difference for βH_o (PCBHO) from OIL sample: OUTWTS vs. PRISM

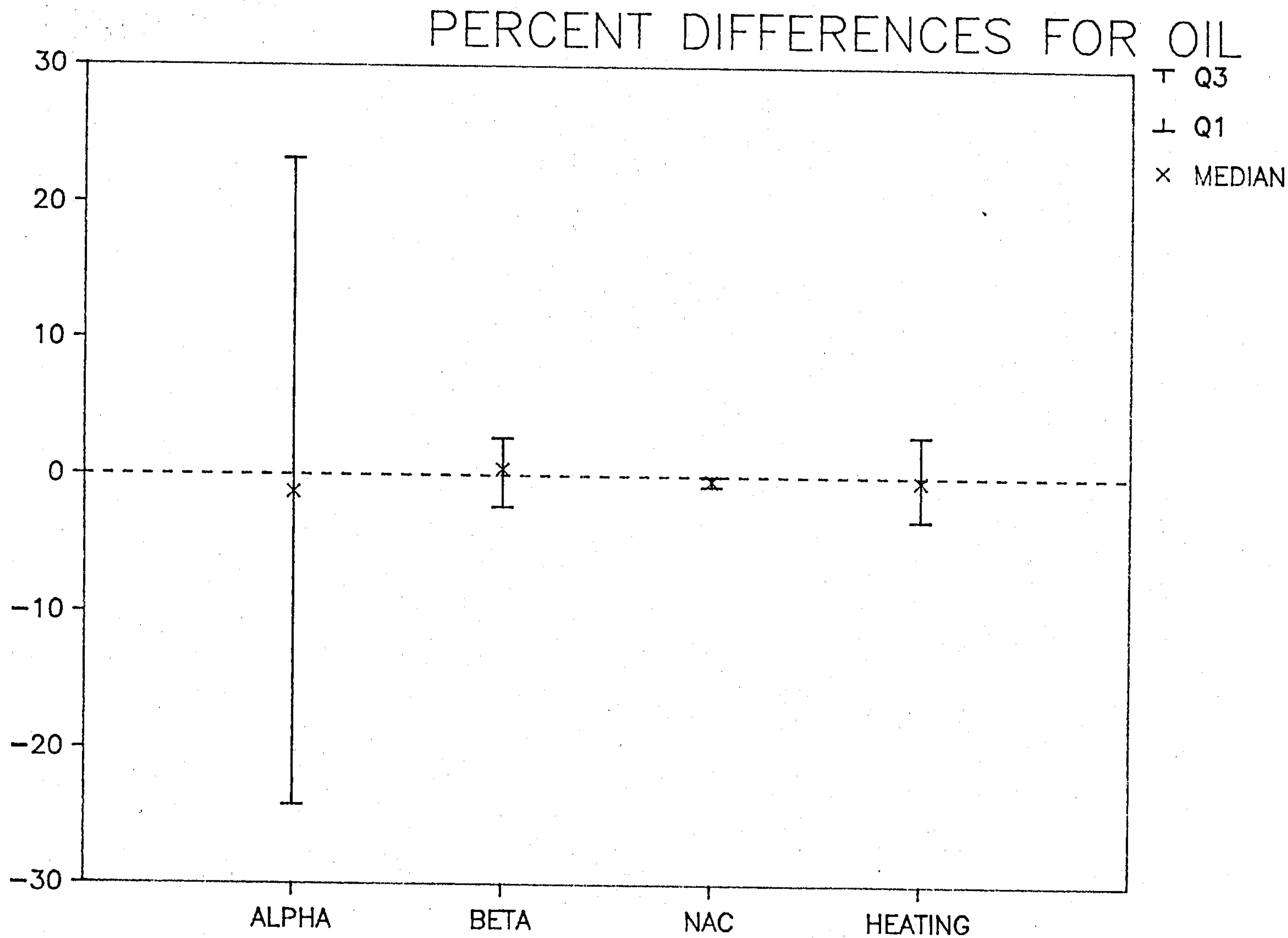


Figure 54. Plot of medians and quartiles of percent differences of PRISM vs. OUTWTS parameters for OIL sample.

```
VARIABLE=CVRALPHA

                 MOMENTS                                          QUANTILES(DEF=4)                        EXTREMES

N                  64  SUM WGTS            64 100% MAX   60.7895      99%      60.7895       LOWEST     HIGHEST
MEAN           2.7178  SUM            173.939  75% Q3     2.0943      95%      5.47176    -0.301887     3.43642
STD DEV       7.48313  VARIANCE       55.9972  50% MED   1.57039      90%      3.06453    -0.264668     4.94709
SKEWNESS       7.6581  KURTOSIS       60.1822  25% Q1    1.31038      10%     0.673779    0.0177985     5.64667
USS           4000.56  CSS            3527.82   0% MIN -0.301887       5%     0.131035     0.470745     8.33333
CV            275.337  STD MEAN      0.935391                          1%    -0.301887        0.625     60.7895
T:MEAN=0      2.90553  PROB>|T|    0.00505294  RANGE     61.0914
SGN RANK         1035  PROB>|S|        0.0001  Q3-Q1    0.783922
NUM ¬= 0           64                          MODE    -0.301887

 MISSING VALUE        .
         COUNT        5
  % COUNT/NOBS     7.25

               HISTOGRAM                    #   BOXPLOT                         NORMAL PROBABILITY PLOT
   62.5+*                                   1      *      62.5+                                                     *
       .                                                      |
       .                                                      |
       .                                                      |
       .                                                      |
       .                                                      |
       .                                                      |
       .                                                      |
       .                                                      |                                                  ++
       .                                                      |                                           +++++++
       .*                                   2      *          |                                    +++++++
       .****************************       59   *--0--*       |                              ++++++      *  *
  -2.5+*                                    2      0      -2.5+  *  *      *********************************
        ----+----+----+----+----+----+                                    +++++++
        * MAY REPRESENT UP TO 2 COUNTS                            +----+----+----+----+----+----+----+----+----+----+
                                                                      -2        -1         0        +1        +2
```

Figure 55. SAS Univariate summary of distribution of CVRs for α (CVRALPHA) from OIL sample. Five homes were eliminated from the sample for this analysis because the standard error of τ was infinite.

```
VARIABLE=CVRBETA

              MOMENTS                                     QUANTILES(DEF=4)                       EXTREMES

N               69  SUM WGTS          69  100% MAX   8.01656      99%    8.01656      LOWEST    HIGHEST
MEAN       1.20212  SUM          82.9459   75% Q3     1.1025      95%    2.25736    0.823023    1.36092
STD DEV   0.910645  VARIANCE    0.829274   50% MED   1.01639      90%    1.29578    0.844678    1.74044
SKEWNESS   6.60389  KURTOSIS     47.8479   25% Q1   0.959803      10%   0.900938    0.848669    2.77431
USS        156.101  CSS          56.3906    0% MIN  0.823023       5%   0.856102    0.863534    3.31891
CV         75.7535  STD MEAN    0.109629                           1%   0.823023    0.892183    8.01656
T:MEAN=0   10.9653  PROB>|T|      0.0001  RANGE      7.19353
SGN RANK    1207.5  PROB>|S|      0.0001  Q3-Q1       0.1427
NUM ¬= 0        69                        MODE      0.823023

STEM LEAF                                  #  BOXPLOT                     NORMAL PROBABILITY PLOT
   8                                       1     *       8.25+                                                  *
   7                                                         |
   7                                                         |
   6                                                     6.75+
   6                                                         |
   5                                                         |
   5                                                     5.25+
   4                                                         |
   4                                                         |
   3                                                     3.75+
   3                                       1     *           |                                       *+++++
   2                                       1     *           |                                   +++*+
   2                                                     2.25+                               ++++++
   1                                       1     *           |                           +++++     *
   1                                      36  *--0--*        |                     *****************
   0                                      29  +-----+    0.75+ *   * * **************
     ----+----+----+----+----+----+----+-                     +----+----+----+----+----+----+----+----+----+----+
                                                                  -2        -1         0        +1        +2
```

Figure 56. SAS Univariate summary of distribution of CVRs for β (CVRBETA) from OIL sample.

VARIABLE=CVRNAC

```
                        MOMENTS                                          QUANTILES(DEF=4)                          EXTREMES

N                 69  SUM WGTS          69  100% MAX   2.21479     99%    2.21479      LOWEST    HIGHEST
MEAN         1.60795  SUM          110.949   75% Q3     1.8388     95%    2.13763    0.908318    2.10902
STD DEV     0.313581  VARIANCE    0.098333   50% MED   1.60609     90%    2.05316    0.948439    2.11321
SKEWNESS  -0.0966207  KURTOSIS   -0.502731   25% Q1    1.37872     10%    1.23305    0.974827    2.16205
USS          185.087  CSS          6.68665    0% MIN  0.908318      5%    1.01604     1.05725    2.18227
CV           19.5019  STD MEAN   0.0377507                          1%   0.908318     1.13566    2.21479
T:MEAN=0      42.594  PROB>|T|      0.0001  RANGE      1.30648
SGN RANK      1207.5  PROB>|S|      0.0001  Q3-Q1     0.460072
NUM ¬= 0          69                        MODE      0.908318

STEM LEAF                    #
  22 1                       1
  21 68                      2
  21 11                      2
  20 56                      2
  20 34                      2
  19 5                       1
  19 0233                    4
  18 559                     3
  18 1233                    4
  17 8                       1
  17 12344                   5
  16 6677                    4
  16 1123                    4
  15 88889                   5
  15 112                     3
  14 68889                   5
  14 034                     3
  13 669                     3
  13 0223                    4
  12 5668                    4
  12 3                       1
  11 5                       1
  11 4                       1
  10 6                       1
  10
   9 57                      2
   9 1                       1
    ----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**-01
```

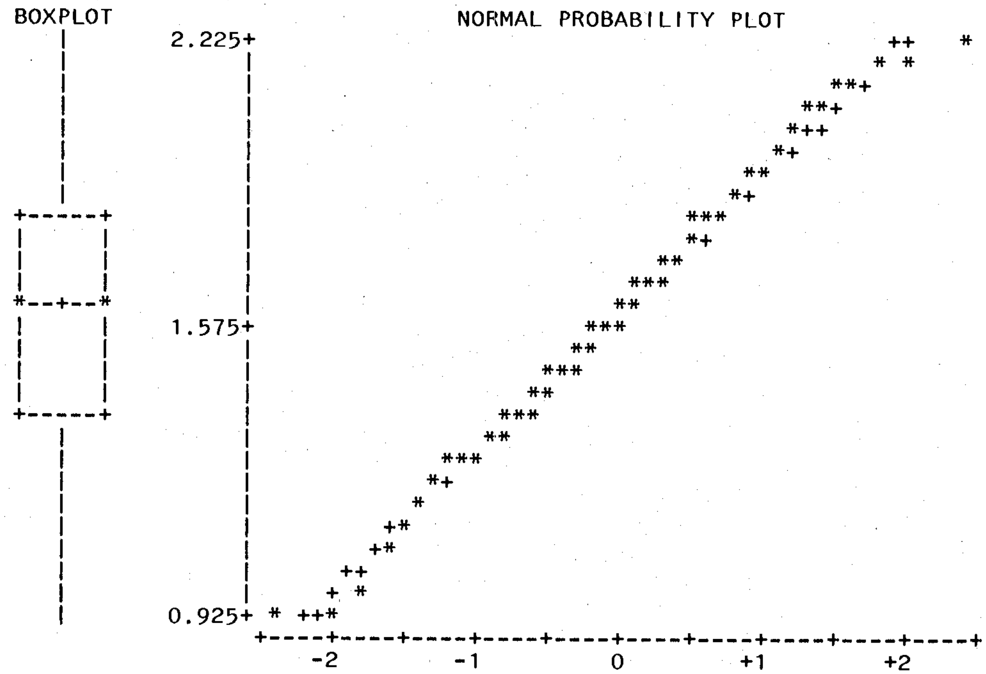


Figure 57. SAS Univariate summary of distribution of CVRs for NAC (CVRNAC) from OIL sample.

```
VARIABLE=CVRBHO

                  MOMENTS                                          QUANTILES(DEF=4)                              EXTREMES

N                   64  SUM WGTS          64  100% MAX     3.3543       99%      3.3543      LOWEST     HIGHEST
MEAN           1.46688  SUM          93.8804   75% Q3     1.71508       95%     2.10408   0.0254649     2.04338
STD DEV        0.46199  VARIANCE    0.213435   50% MED    1.43504       90%     2.00317    0.659767     2.06821
SKEWNESS      0.685592  KURTOSIS       4.281   25% Q1     1.13681       10%     1.02434    0.894954     2.11604
USS            151.158  CSS          13.4464    0% MIN  0.0254649        5%    0.899775    0.914239     2.33393
CV             31.4947  STD MEAN   0.0577488                             1%   0.0254649    0.934666      3.3543
T:MEAN=0       25.4011  PROB>|T|      0.0001  RANGE       3.32883
SGN RANK          1040  PROB>|S|      0.0001  Q3-Q1      0.578269
NUM ¬= 0            64                        MODE      0.0254649

 MISSING VALUE        .
         COUNT        5
  % COUNT/NOBS     7.25

STEM LEAF                        #  BOXPLOT                        NORMAL PROBABILITY PLOT
  32 5                           1     0        3.3+                                                    *
  30                                               |
  28                                               |
  26                                               |                                                      +
  24                                               |                                                  ++++
  22 3                           1     |           |                                             +++++*
  20 1472                        4     |           |                                         ++*****
  18 670099                      6     |           |                                     +*****
  16 22445902459                11  +-----+     1.7+                               +*******
  14 033477016899               12  *--+--*        |                           *****
  12 0288935779                 10  |     |        |                       *****
  10 23467911334467             14  +-----+        |              *********
   8 913                         3     |           |           ***++++
   6 6                           1     |           |         * ++++
   4                                               |     +++++
   2                                               |++
   0 3                           1     0        0.1+  *
     ----+----+----+----+                           +----+----+----+----+----+----+----+----+----+----+
  MULTIPLY STEM.LEAF BY 10**-01                           -2        -1         0        +1        +2
```

Figure 58. SAS Univariate summary of distribution of CVRs for βH_o (CVRBHO) from OIL sample. Five homes were eliminated from the sample for this analysis because the standard error of τ was infinite.

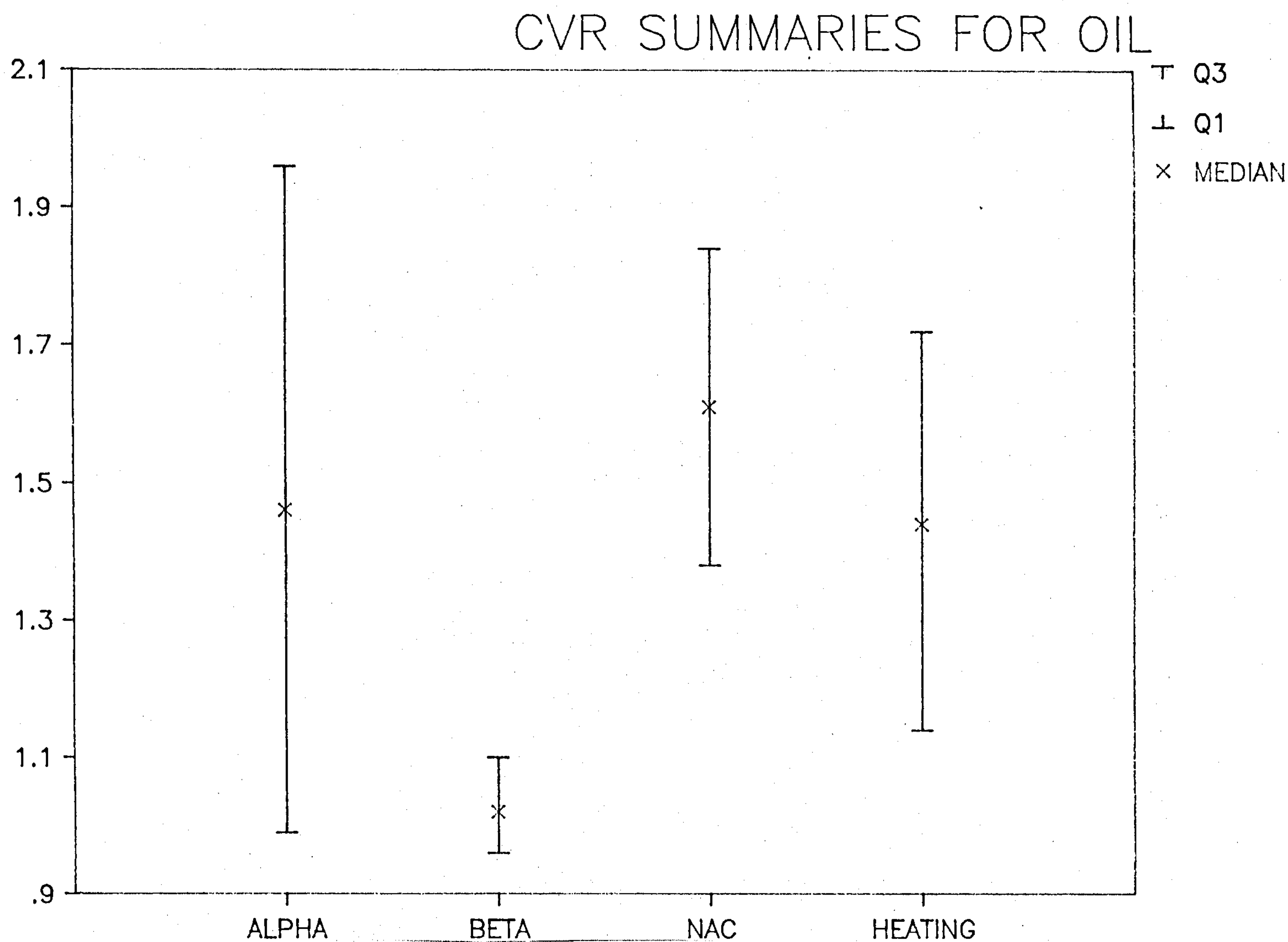


Figure 59. Plot of medians and quartiles of CVRs of PRISM vs. OUTWTS parameters for OIL sample. For CVR(α), only the 30 HW houses with positive α and finite se(α) for both PRISM and OUTWTS were included in the analysis.

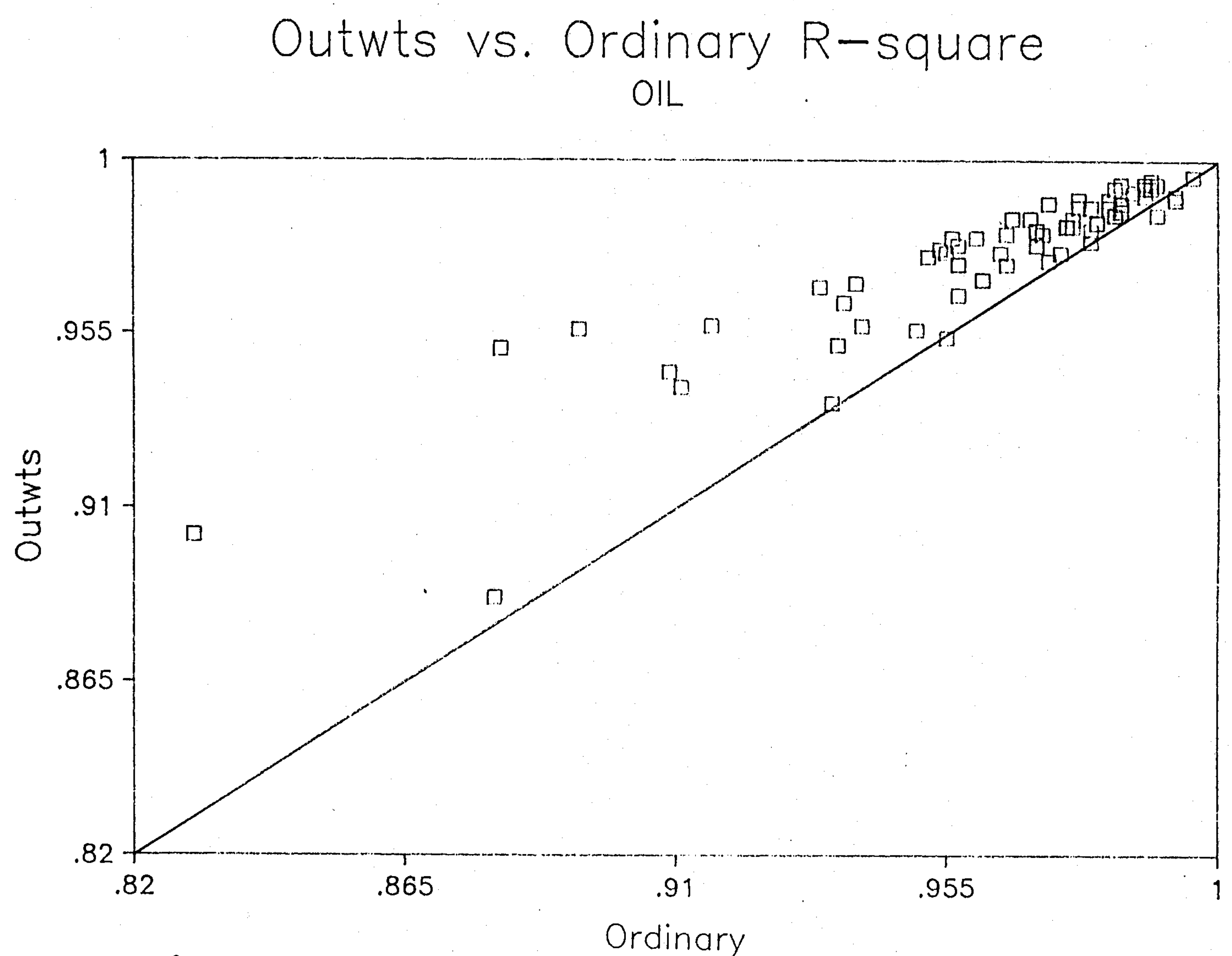


Figure 60. R^2 estimates for Weighted (OUTWTS) vs. Ordinary PRISM for OIL sample.

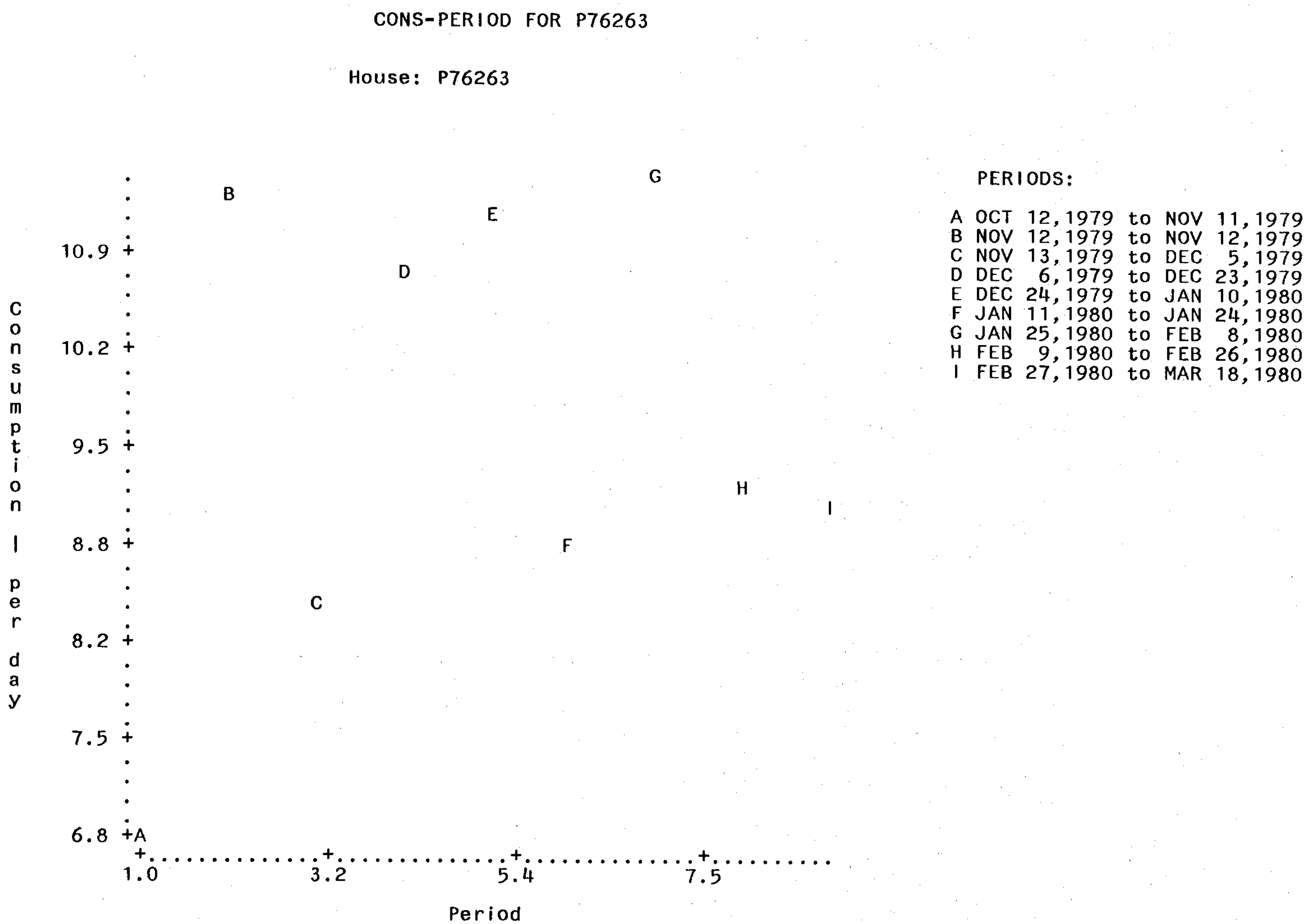
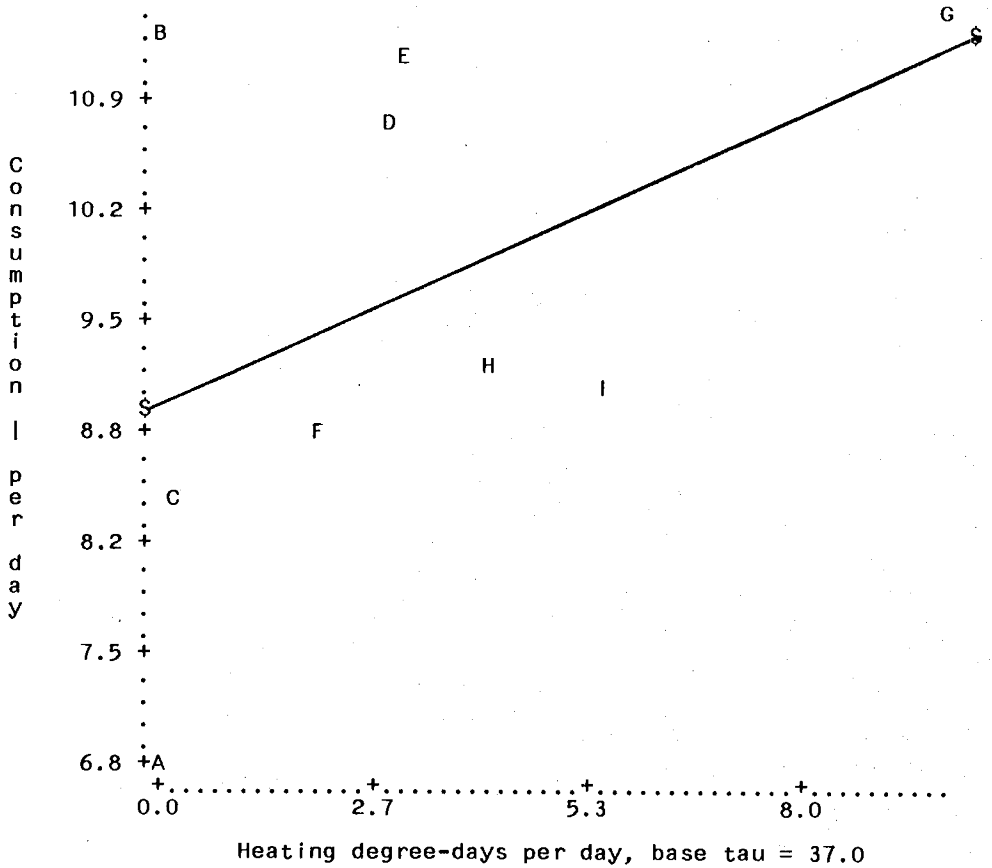


Figure 61. Plot of consumption vs. period for House P76263 from OIL sample.

CONS-HDD FOR P76263, PRISM

House:P76263 ,alpha= 8.98,beta= 0.22,R2= 0.2049



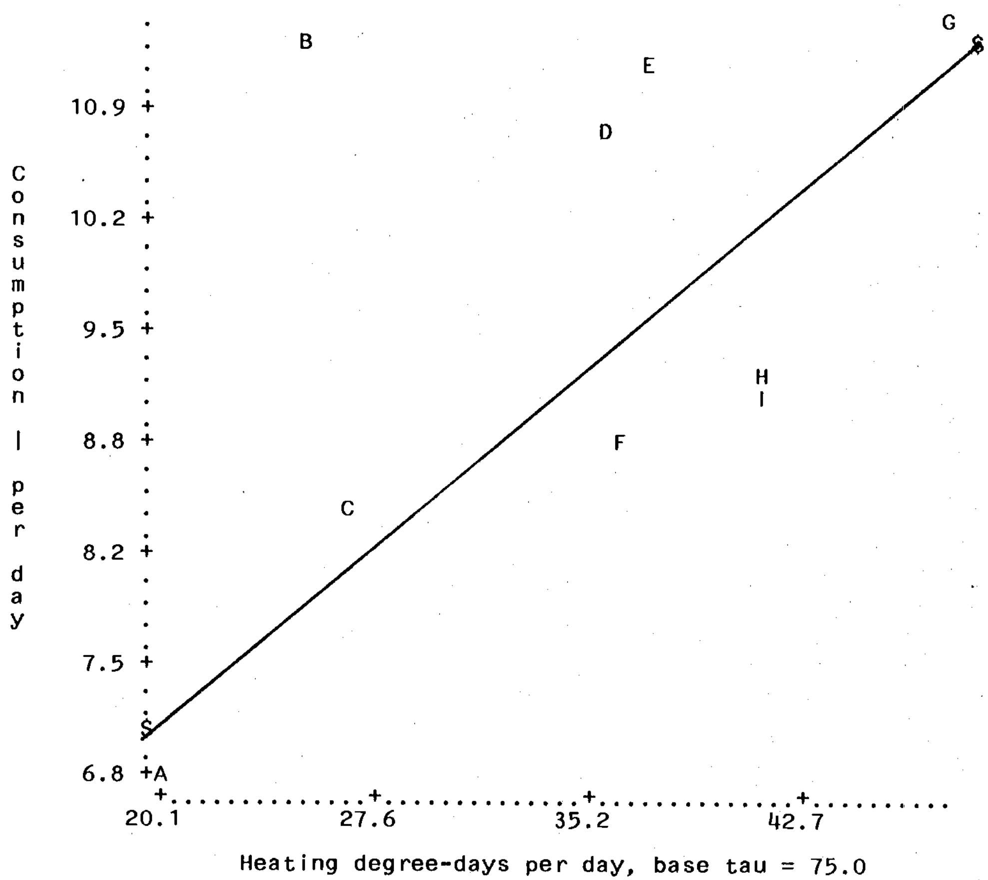
PERIODS:

A OCT 12,1979 to NOV 11,1979
B NOV 12,1979 to NOV 12,1979
C NOV 13,1979 to DEC 5,1979
D DEC 6,1979 to DEC 23,1979
E DEC 24,1979 to JAN 10,1980
F JAN 11,1980 to JAN 24,1980
G JAN 25,1980 to FEB 8,1980
H FEB 9,1980 to FEB 26,1980
I FEB 27,1980 to MAR 18,1980

Figure 62. PRISM plot of consumption vs. heating degree-days for House P76263 from OIL sample.

CONS-HDD FOR P76263, OUTWTS

House:P76263 ,alpha= 4.43,beta= 0.14,R2= 0.6546



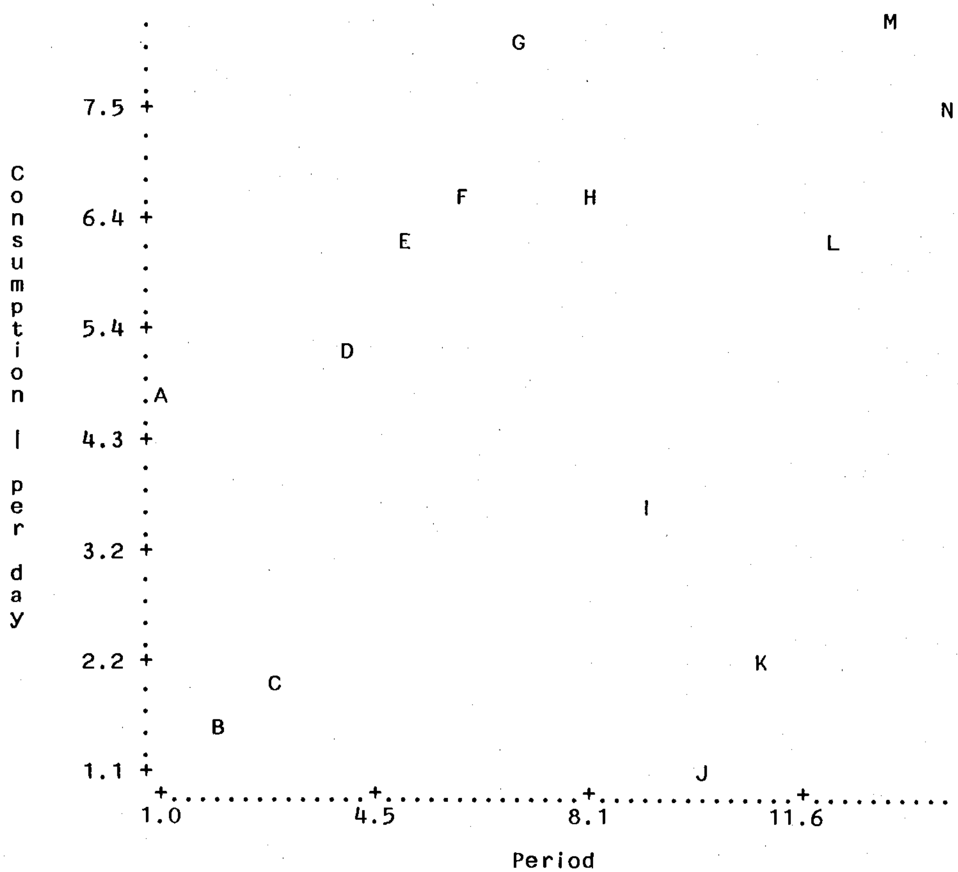
PERIODS:

A OCT 12,1979 to NOV 11,1979
B NOV 12,1979 to NOV 12,1979
C NOV 13,1979 to DEC 5,1979
D DEC 6,1979 to DEC 23,1979
E DEC 24,1979 to JAN 10,1980
F JAN 11,1980 to JAN 24,1980
G JAN 25,1980 to FEB 8,1980
H FEB 9,1980 to FEB 26,1980
I FEB 27,1980 to MAR 18,1980

Figure 63. OUTWTS plot of consumption vs. heating degree-days for House P76263 from OIL sample.

CONS-PERIOD FOR P57490

House: P57490



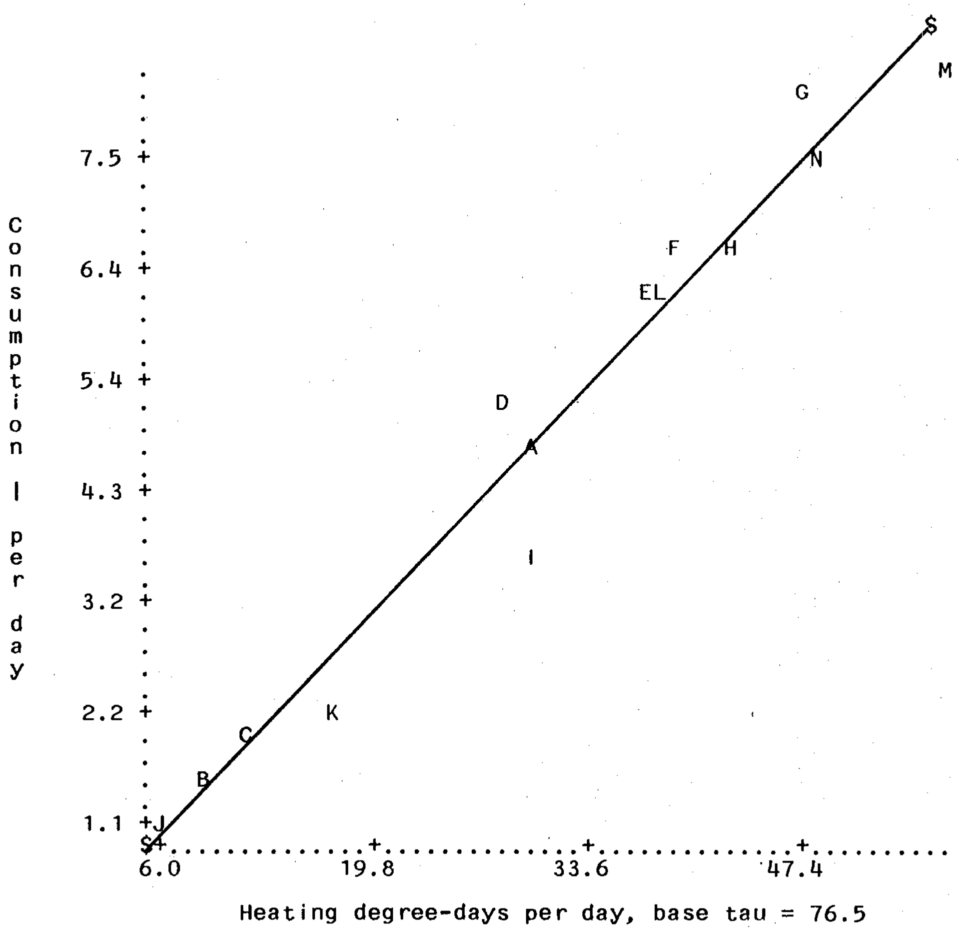
PERIODS:

A	MAR 2,1979	to	APR 9,1979
B	APR 10,1979	to	AUG 12,1979
C	AUG 13,1979	to	NOV 4,1979
D	NOV 5,1979	to	DEC 9,1979
E	DEC 10,1979	to	JAN 4,1980
F	JAN 5,1980	to	JAN 21,1980
G	JAN 22,1980	to	FEB 12,1980
H	FEB 13,1980	to	MAR 10,1980
I	MAR 11,1980	to	APR 22,1980
J	APR 23,1980	to	AUG 27,1980
K	AUG 28,1980	to	NOV 18,1980
L	NOV 19,1980	to	DEC 18,1980
M	DEC 19,1980	to	JAN 12,1981
N	JAN 13,1981	to	FEB 5,1981

Figure 64. Plot of consumption vs. period for House P57490 from OIL sample.

CONS-HDD FOR P57490, PRISM

House:P57490 ,alpha= 0.02,beta= 0.16,R2= 0.9654



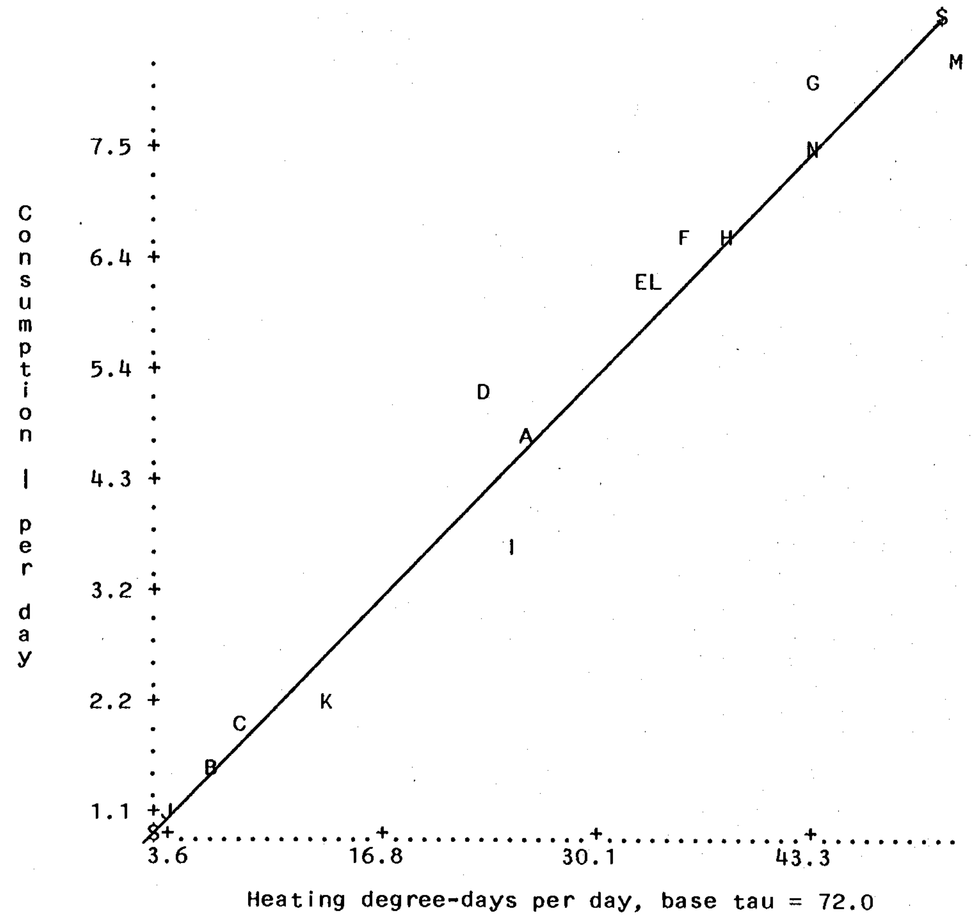
PERIODS:

A	MAR 2,1979	to	APR 9,1979
B	APR 10,1979	to	AUG 12,1979
C	AUG 13,1979	to	NOV 4,1979
D	NOV 5,1979	to	DEC 9,1979
E	DEC 10,1979	to	JAN 4,1980
F	JAN 5,1980	to	JAN 21,1980
G	JAN 22,1980	to	FEB 12,1980
H	FEB 13,1980	to	MAR 10,1980
I	MAR 11,1980	to	APR 22,1980
J	APR 23,1980	to	AUG 27,1980
K	AUG 28,1980	to	NOV 18,1980
L	NOV 19,1980	to	DEC 18,1980
M	DEC 19,1980	to	JAN 12,1981
N	JAN 13,1981	to	FEB 5,1981

Figure 65. PRISM plot of consumption vs. heating degree-days for House P57490 from OIL sample.

CONS-HDD FOR P57490, OUTWTS

House:P57490 ,alpha= 0.45,beta= 0.16,R2= 0.9727



PERIODS:

A MAR 2,1979 to APR 9,1979
B APR 10,1979 to AUG 12,1979
C AUG 13,1979 to NOV 4,1979
D NOV 5,1979 to DEC 9,1979
E DEC 10,1979 to JAN 4,1980
F JAN 5,1980 to JAN 21,1980
G JAN 22,1980 to FEB 12,1980
H FEB 13,1980 to MAR 10,1980
I MAR 11,1980 to APR 22,1980
J APR 23,1980 to AUG 27,1980
K AUG 28,1980 to NOV 18,1980
L NOV 19,1980 to DEC 18,1980
M DEC 19,1980 to JAN 12,1981
N JAN 13,1981 to FEB 5,1981

Figure 66. OUTWTS plot of consumption vs. heating degree-days for House P57490 from OIL sample.

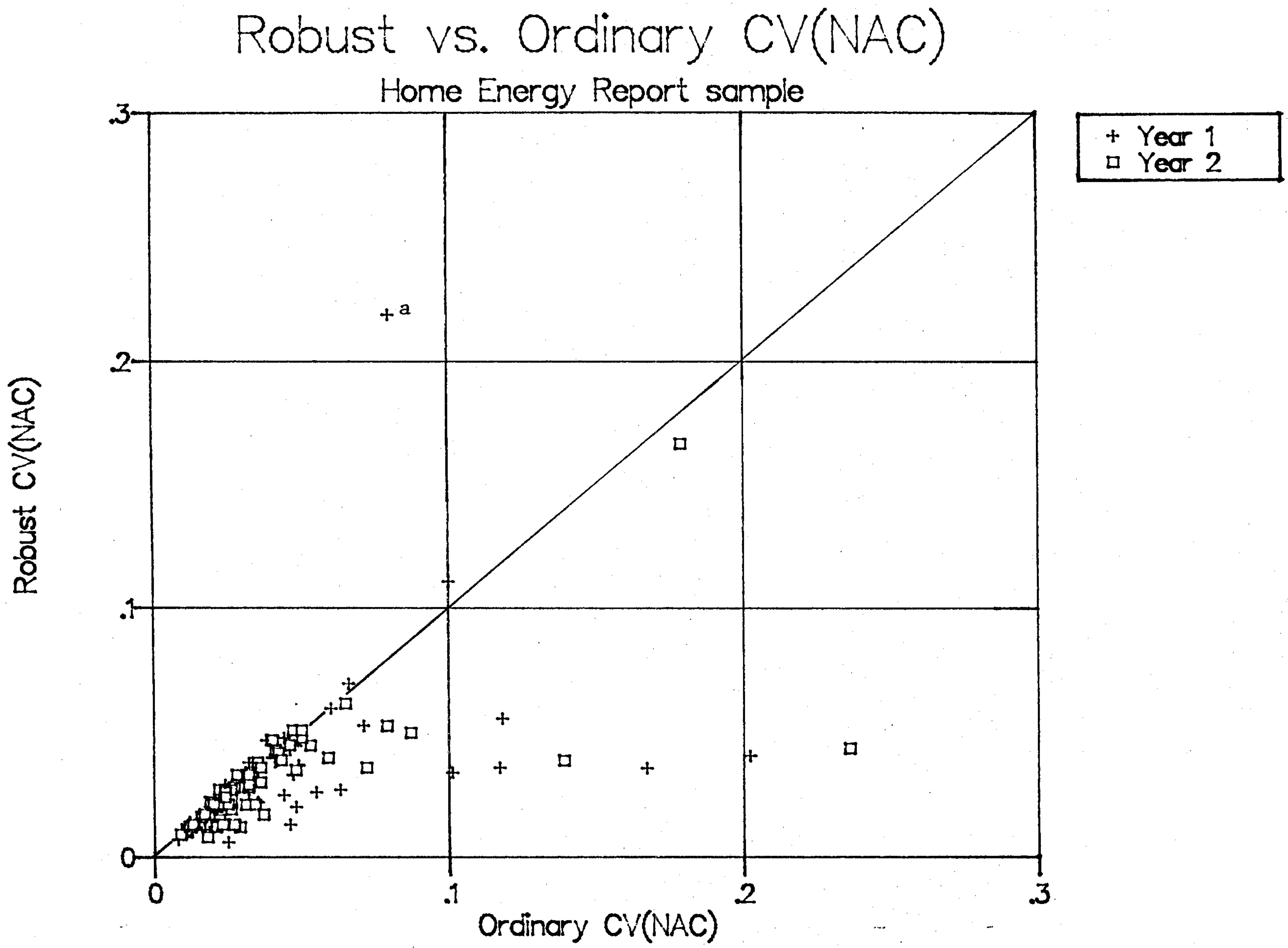


Figure 67. CV(NAC) for Robust (RPRISM) vs. Ordinary PRISM for Home Energy Report sample.

[a] This case, showing a large increase in CV(NAC), is from the set of houses preselected for probable data anomalies. House was without electric heating, and had extremely low R^2 (<0.2) with and without using Robust PRISM.

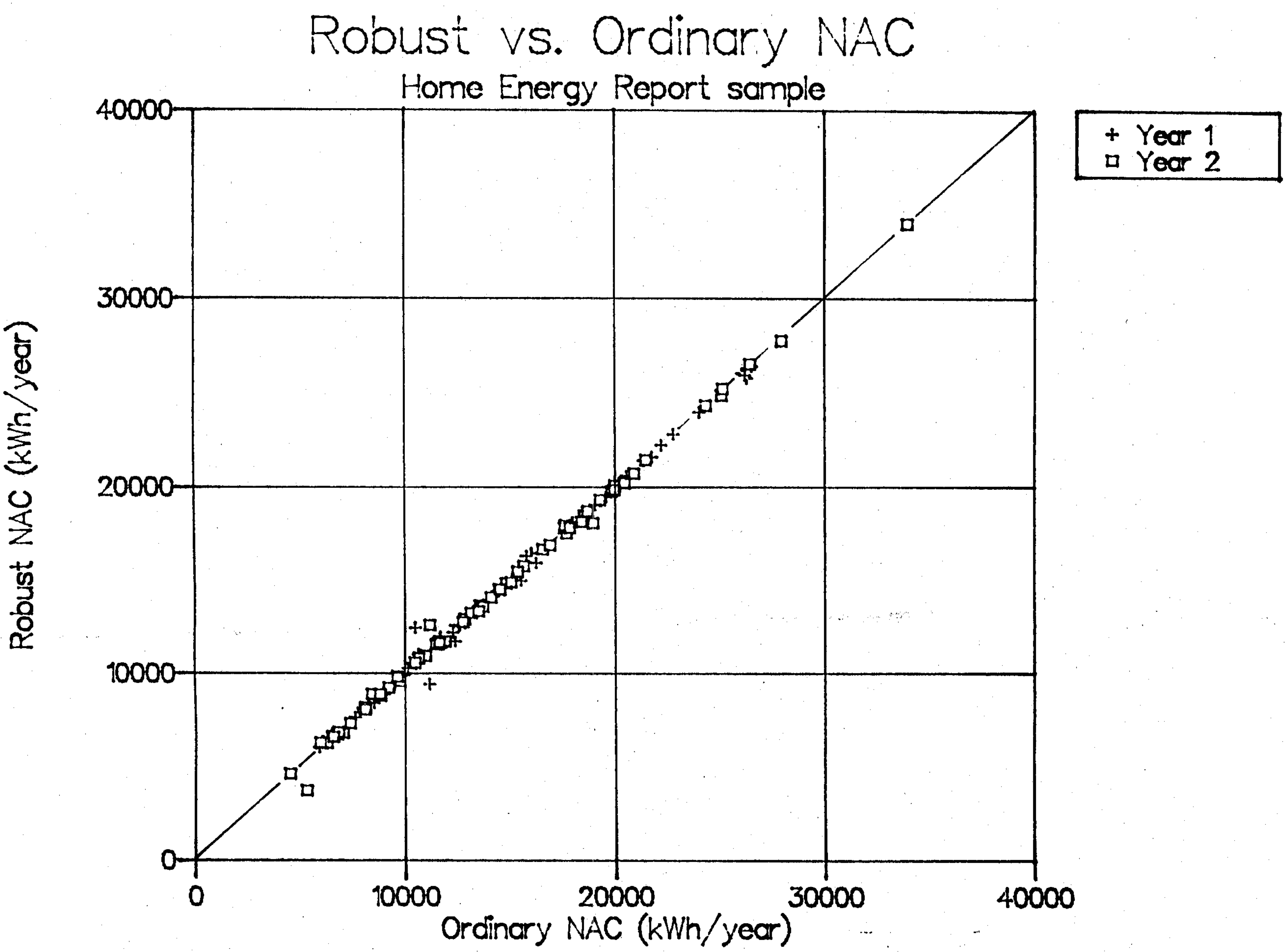


Figure 68. NAC estimates for Robust (RPRISM) vs. Ordinary PRISM for Home Energy Report sample.